



## EMERGENCE OF SCHOOL STUDENTS' MATHEMATICAL PROOF ACTIVITIES IN GEOMETRIC SEQUENCE LEARNING: A THEORY OF DIDACTICAL SITUATIONS PERSPECTIVE

Mohamad Gilar Jatisunda<sup>1\*</sup>, Wulan Putri Utami<sup>2</sup>  
Department of Mathematics Education, Universitas Majalengka, Indonesia  
[g.jatisunda@unma.ac.id](mailto:g.jatisunda@unma.ac.id), [wp.utami@gmail.com](mailto:wp.utami@gmail.com)

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### ***Corresponding Author:***

Mohamad Gilar Jatisunda  
Universitas Majalengka  
Jl. KH. Abdul Halim No. 103, Majalengka Kulon,  
Kec. Majalengka, Kabupaten Majalengka, Jawa Barat 45418.  
[sg.jatisunda@unma.ac.id](mailto:sg.jatisunda@unma.ac.id)  
Contact Person: 0812-2240-180

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### ***ABSTRACT***

Mathematical proof plays an essential role in mathematics education because it supports the development of reasoning, communication, and conceptual understanding. However, students frequently encounter difficulties in constructing and validating mathematical arguments, often relying on procedural operations rather than explicit mathematical justification. Existing studies have predominantly examined proof from cognitive and epistemological perspectives, whereas comparatively little attention has been paid to understanding how proof-related activities emerge through interactions within didactical situations. This study therefore investigated how school students constructed and validated mathematical reasoning during geometric sequence learning through the lens of the Theory of Didactical Situations (TDS). A qualitative interpretive case study design was employed involving 36 eleventh-grade students organized into six collaborative groups. Data were collected through classroom observations, students' written productions, semi-structured interviews, and field notes. Learning activities were implemented in accordance with the phases of TDS: action, formulation, validation, and institutionalization. Data were analyzed using iterative qualitative procedures involving data reduction, data display, and conclusion drawing. The findings revealed recurring characteristics across the four phases. During the action phase, students primarily initiated problem-solving through the symbolic transformation of numerical information. During the formulation phase, students relied predominantly on formula-based representations and procedural communication. Validation activities were characterized mainly by numerical agreement and procedural completion rather than by inferential justification. Furthermore, the institutionalization phase demonstrated the emergence of shared procedural structures across groups. The findings indicate that proof-related activities emerged not solely from individual reasoning processes but also through interactions among students, symbolic representations, and the didactical milieu. This study contributes to mathematics education research by integrating perspectives on mathematical proof with the Theory of Didactical Situations and by conceptualizing proof activity as a didactically situated phenomenon rather than exclusively as an individual cognitive process.

**Keywords:** mathematical proof; Theory of Didactical Situations; proof validation; geometric sequence; school students; didactical milieu

### **INTRODUCTION**

Mathematical proof occupies a central position in mathematics because it serves not only to establish the validity of mathematical statements but also to support the development of mathematical understanding, communication, and reasoning. Through proof, learners are expected to justify why

mathematical relationships hold under general conditions rather than relying on isolated cases or procedural calculations. Consequently, proof is widely considered an essential component of mathematics learning because it promotes the transition from procedural execution toward deeper conceptual understanding and meaningful mathematical argumentation (Hanna, 2002, 2020; Stylianides et al., 2024). Within school mathematics, proof therefore functions not merely as a formal verification process but also as a means for constructing mathematical meaning.

Despite its recognized importance, mathematical proof remains one of the most challenging aspects of mathematics learning. Previous studies have consistently reported that students often encounter difficulties in constructing and understanding mathematical arguments because they tend to emphasize procedural operations and symbolic manipulation rather than mathematical justification (Knuth et al., 2019; Ahmadpour et al., 2019). Students may frequently arrive at correct answers through algorithmic procedures while demonstrating limited understanding of why such procedures work mathematically. This distinction is theoretically important because procedural correctness does not necessarily imply conceptual understanding. As proposed by Skemp (1976), students may develop an instrumental understanding characterized by rule application without establishing meaningful conceptual relationships, whereas proof requires a relational understanding that involves coherent mathematical structures.

Research on mathematical proof has predominantly examined cognitive and epistemological dimensions, including proof schemes, proof comprehension, and proof validation processes. Existing studies indicate that students employ diverse criteria when accepting or rejecting mathematical arguments, including logical structure, prior knowledge, and perceived mathematical validity (Stylianides & Stylianides, 2009; Kirsten & Greefrath, 2023). Furthermore, students' judgments about mathematical arguments are often shaped by authority, familiar procedures, and socially accepted classroom practices rather than by purely deductive reasoning (Bleiler-Baxter et al., 2023). From a sociocultural perspective, mathematical activity cannot be understood solely as an individual cognitive process because mathematical meanings are negotiated through interactions and sociomathematical norms established within learning environments (Lerman, 1998; Stephan, 2020).

A complementary perspective for understanding these interactions is provided by the Theory of Didactical Situations (TDS). According to TDS, mathematical knowledge emerges through interactions among learners, teachers, mathematical tasks, and the didactical milieu within which learning occurs (Brousseau & Warfield, 2020). Learning is conceptualized as a dynamic process involving phases of action, formulation, validation, and institutionalization that progressively shape students' mathematical meanings. Previous studies have suggested that characteristics of the didactical milieu may substantially influence students' opportunities to formulate conjectures, construct arguments, and engage in proof-related activities (Artigue, 2014; Bloch, 2003). Therefore, students' written productions may be

interpreted not merely as products of internal cognitive processes but also as manifestations of interactions occurring within particular didactical situations.

These issues become particularly relevant within mathematical topics involving recursive and multiplicative reasoning, such as geometric sequences. Studies on geometric sequences have shown that students frequently encounter learning obstacles related to generalization, recursive reasoning, and symbolic representation (Anzani et al., 2022; Magfiroh et al., 2024). Similarly, students' reasoning in sequence contexts has often been characterized by procedural tendencies that limit opportunities for deeper mathematical explanation and proof construction (Faridah & Marlina, 2025). However, previous studies on mathematical proof have predominantly focused on students' cognitive characteristics and individual reasoning processes, whereas studies employing TDS have primarily concentrated on describing didactical phases and classroom interactions. Comparatively little attention has been paid to understanding how proof-related activities themselves emerge from interactions within didactical situations. Consequently, an important conceptual gap remains concerning the relationship between students' validation processes and the didactical conditions under which mathematical arguments are constructed.

The present study addresses this limitation by integrating the Theory of Didactical Situations with school students' perspectives on mathematical proof. Rather than examining students' written productions solely in terms of procedural correctness or cognitive performance, this study conceptualizes proof-related activity as a phenomenon emerging from interactions among students, mathematical tasks, and the didactical milieu. The novelty of this study lies in three aspects: first, it integrates TDS with perspectives on mathematical proof within school mathematics contexts; second, it conceptualizes proof activity as a didactically situated process rather than solely an individual cognitive phenomenon; and third, it provides empirical evidence regarding how students' validation processes are manifested across phases of action, formulation, validation, and institutionalization during geometric sequence learning. Accordingly, this study aims to investigate how school students construct and validate mathematical reasoning in didactic contexts while learning geometric sequences.

## **METHODS**

### **Research Design**

This study employed a qualitative, interpretive case study design grounded in the Theory of Didactical Situations (TDS) and in school students' perspectives on mathematical proof to investigate how mathematical reasoning and proof-related activities emerged during learning about geometric sequences. A qualitative interpretive approach was adopted because the purpose of the study was not to determine causal relationships or measure learning outcomes statistically but rather to understand how students constructed and validated mathematical arguments through interactions with mathematical



tasks and the didactical milieu. Within this study, proof was not viewed exclusively as a formal deductive product but as a process through which students generated, communicated, justified, and validated mathematical ideas throughout learning activities. The Theory of Didactical Situations, proposed by Brousseau (2002), served as the primary framework for analyzing students' mathematical activities across the phases of action, formulation, validation, and institutionalization. These stages were employed to examine how students interacted with mathematical tasks and the classroom milieu during learning. To provide a more detailed analysis of students' reasoning processes, perspectives on school students' mathematical proof were integrated into the analysis, particularly those related to proof validation, acceptance criteria, and mathematical justification (Stylianides et al., 2024). The integration of these perspectives enabled proof activity to be interpreted as an emergent didactical phenomenon rather than solely an individual cognitive process.

### **Participants and Research Context**

The participants in this study were 36 eleventh-grade students from a senior high school in Indonesia enrolled in a mathematics course on geometric sequences. Participants were selected using purposive sampling because the study required students who had not yet received formal instruction in geometric sequences, thereby allowing mathematical ideas and validation processes to emerge more naturally within the didactical situations. The participant selection process was conducted in collaboration with the classroom teacher, based on instructional records and learning progress. Students meeting these criteria were then organized into six collaborative groups consisting of six students each to facilitate interaction, discussion, and collective knowledge construction during learning activities. The study was conducted during regular classroom instruction on geometric sequences because the topic requires recursive and multiplicative reasoning processes, including pattern recognition, generalization, and mathematical relationships. Previous studies have shown that geometric sequences frequently pose conceptual difficulties for students' interpretation of multiplicative structures and contextual representations (Magfiroh et al., 2024). Therefore, learning geometric sequences provides an appropriate context for investigating how mathematical arguments and validation processes emerge during classroom interactions. Within the learning process, the researcher simultaneously served as a teacher-researcher, responsible for designing and implementing didactic situations. This dual role aligns with the didactical perspective, in which learning situations are intentionally designed to create interactions between students and the milieu that support knowledge construction (Artigue, 2014; Suryadi, 2019).

### **Data Sources and Research Instruments**

Data were collected from multiple sources to provide a comprehensive understanding of students' proof-related activities within the didactical situations. The primary data sources included classroom observations, students' written responses on group worksheets, semi-structured interviews, and field



notes. Classroom observations were used to document students' activities and interactions across the phases of the Theory of Didactical Situations: action, formulation, validation, and institutionalization. Particular attention was given to students' exploration of mathematical ideas, communication patterns, and validation processes occurring during collaborative activities. Students' written productions constituted the principal source of analysis because they represented visible manifestations of students' mathematical reasoning processes. The worksheets were designed to elicit mathematical activity involving geometric sequence concepts in contextual situations that require students to explain, justify, and validate their solutions. Semi-structured interviews were conducted with six selected students, one student from each group. Participant selection for interviews was based on differences in written response characteristics and observed reasoning patterns during classroom activities. The interviews aimed to examine students' thinking processes, justification strategies, and criteria used when accepting mathematical arguments. Field notes were used to document contextual information and classroom interactions that were not fully represented in written responses and observation records. To ensure the suitability of all instruments for the study objectives, instrument validation was conducted through expert judgment involving one mathematics education lecturer and one experienced mathematics teacher. The validation process focused on content relevance, conceptual alignment, and language clarity, following procedures used in previous didactical studies.

### **Research Procedures**

The learning activities were implemented in accordance with the phases of the Theory of Didactical Situations proposed by Brousseau (2002), namely action, formulation, validation, and institutionalization. During the action phase, students were introduced to contextual problems involving geometric sequences and encouraged to explore possible strategies independently without explicit procedural guidance from the teacher. The formulation phase involved collaborative discussions in which students communicated ideas, constructed representations, and developed initial mathematical explanations. The validation phase emphasized evaluating and justifying mathematical solutions through student interaction. During this phase, students compared alternative approaches, discussed differences in reasoning, and attempted to establish the validity of their solutions. Finally, the institutionalization phase involved formalizing students' constructed ideas by connecting their findings to formal mathematical concepts related to geometric sequences. Following the learning activities, students completed worksheet tasks designed to capture proof-related activity and mathematical reasoning. Semi-structured interviews were subsequently conducted with selected students to gain a deeper understanding of their reasoning and validation processes.

### **Data Analysis**

Data analysis followed the qualitative analysis procedures proposed by Miles and Huberman (1992), consisting of data reduction, data display, and conclusion drawing. During the data reduction

stage, observation data, students' written productions, interview transcripts, and field notes were selected and organized according to the phases of the Theory of Didactical Situations and proof-related analytical dimensions. The analysis was conducted at two interconnected levels. At the macro level, students' activities were examined according to the TDS phases of action, formulation, validation, and institutionalization. At the micro level, students' mathematical arguments were analyzed from a proof perspective, focusing on logical structures, conceptual explanations, and validation processes. Particular attention was given to how students constructed inferential relationships, justified mathematical claims, and accepted mathematical arguments during classroom interactions. To strengthen credibility and trustworthiness, triangulation was achieved by comparing information from classroom observations, students' written work, interviews, and field notes. This procedure enabled the examination of the consistency of interpretations across multiple sources of evidence and ensured that the findings accurately reflected students' mathematical activities within the didactical situations.

### **Ethical Considerations**

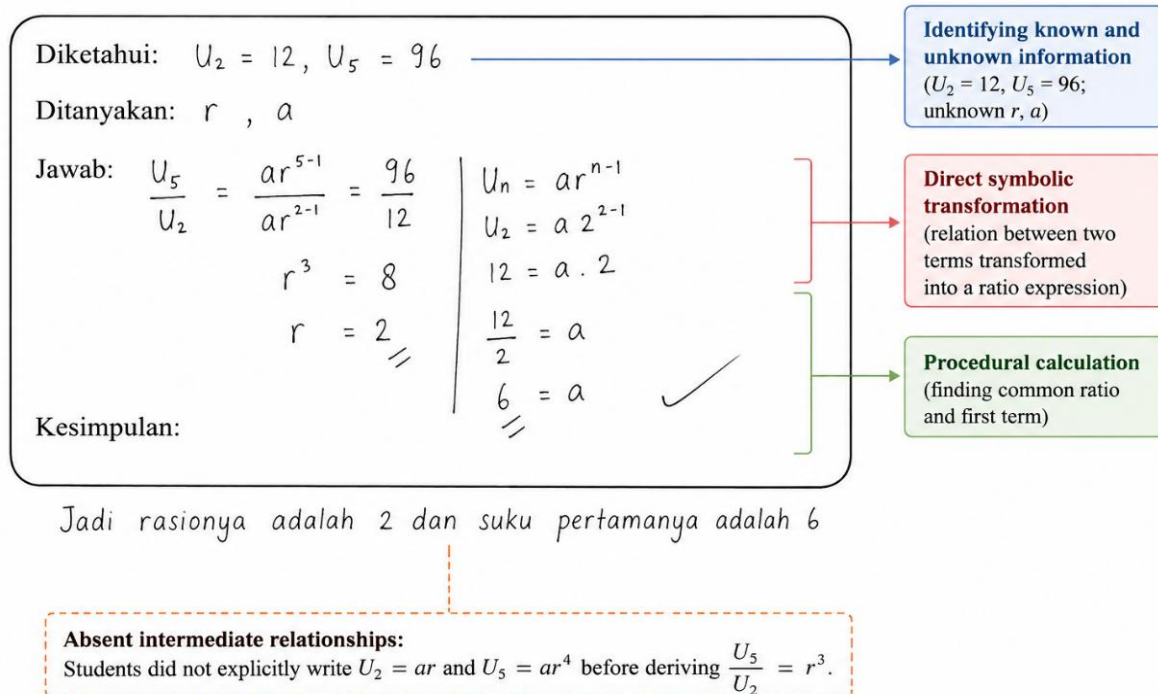
This study adhered to established ethical principles governing educational research involving human participants. Participants were informed of the study's purpose and procedures, and that participation was entirely voluntary. Informed consent was obtained prior to data collection. To protect participants' privacy, responses were collected anonymously, and all data were treated confidentially. Furthermore, the study ensured that participation involved no foreseeable physical or psychological risks.

## **FINDINGS**

### **Students' Mathematical Activities During the Action Phase**

During the action phase, students initially interacted with contextual geometric sequence tasks by attempting to identify relevant information embedded within the problem situations. Classroom observations revealed that most groups demonstrated highly similar initial responses in approaching the task. Rather than first discussing the contextual relationships in the problem, students immediately focused on extracting numerical information to be transformed into symbolic expressions for subsequent calculations. The initial interaction with the task, therefore, appeared to emphasize recognition of the operational elements required to complete the problem. Across groups, students generally began by separating known and unknown quantities and assigning symbolic labels to these elements. Known values such as  $U_2 = 12$  and  $U_5 = 96$  were first identified, followed by attempts to determine the variables required for solution procedures, particularly the common ratio ( $r$ ) and the first term ( $a$ ). Observational data further indicated that students rarely discussed the meaning of these quantities in the context itself. Instead, attention was directed toward determining which mathematical components could be used immediately in formula-based procedures. Figure 1 presents a representative

example of students' written production during the action phase. The written response illustrates how students initially organized information by identifying known quantities and directly converting them into symbolic expressions.



**Figure 1.** Students' written production illustrating direct symbolic transformation during the action phase.

The written production demonstrates that students immediately moved toward procedural symbolic operations by relating the two known terms and calculating the common ratio. However, the response does not explicitly explain how the relationship between the terms was established mathematically. Intermediate relationships connecting the sequence terms to the general form of geometric sequences were not expressed in the written work. Instead, students directly proceeded from identifying known quantities to symbolic manipulation. Interview data further supported this pattern. When asked how the relationship between  $U_2$  and  $U_5$  was established, one student explained:

*Interviewer: Why did you directly divide  $U_5$  by  $U_2$ ?*

*Student G2: Because we usually compare the terms directly to find the ratio. We remembered the formula and immediately used it.*

The response suggests that students primarily relied on memorized procedures rather than on constructing explicit mathematical relationships between terms in the sequence. Similar patterns were observed across several groups. Students frequently initiated problem-solving activities by extracting numerical information and applying symbolic procedures directly. This recurring pattern indicates that

initial engagement with the task emphasized symbolic execution and procedural action rather than exploration of mathematical relationships embedded within the problem situation.

### Students' Mathematical Activities During the Formulation Phase

During the formulation phase, students communicated their ideas within groups and attempted to construct mathematical representations to explain their solution strategies. Classroom observations revealed that students increasingly employed formal symbolic expressions and formulas to represent geometric relationships during collaborative discussions. Rather than relying solely on contextual descriptions, students gradually transformed their reasoning into mathematical forms that could support subsequent calculations.

Figure 2 presents a representative example of students' written production during the formulation phase. The response illustrates how students translated contextual information into generalized symbolic representations using the geometric sequence formula:

$$U_n = ar^{n-1}$$

The figure shows a handwritten student solution for a problem involving a geometric sequence. The solution is annotated with four colored boxes on the right side, each pointing to a specific part of the work:

- Contextual information** (blue box): Points to the given information: "Diketahui:  $r = 4$ ,  $6 + u_3 = 0$  10 20  $u_2 = 64$ ". The terms 10, 20, and  $u_2 = 64$  are grouped together with a bracket and labeled  $u_1$ ,  $u_2$ , and  $u_3$  respectively.
- Symbolic representation** (green box): Points to the general formula: "Jawab:  $U_n = ar^{n-1}$ ".
- Procedural computation** (red box): Points to the calculation steps: " $u_3 = a \cdot r^{3-1}$ ", " $64 = a \cdot 4^2$ ", and " $a = \frac{64}{16} = 4$ ". A red checkmark is placed next to the final calculation.
- Conclusion** (purple box): Points to the final statement: "Kesimpulan: Jadi, jumlah bakteri mula-mula adalah 4".

Below the main solution, there is a dashed box titled "Incomplete mathematical explanation" which contains the text: "Intermediate relationships connecting specific terms to the general form are not explicitly stated." This box is divided into two parts:

- Omitted relationships:** " $u_2 = ar$ " and " $u_5 = ar^4$ ".
- Students directly proceed to:** " $\frac{u_5}{u_2} = r^3$ ".

An arrow points from the omitted relationships to the direct step, indicating that the intermediate steps were skipped.

**Figure 2.** Students' written production illustrating symbolic representation during the formulation phase

The written production demonstrates that students successfully transformed contextual information into mathematical expressions. However, explanations accompanying these symbolic representations remained comparatively limited. Students frequently presented formulas and computational procedures without explicitly explaining how the relationships among sequence terms were established mathematically. For example, students often proceeded directly to symbolic transformations involving ratio relationships without expressing intermediate relationships that connect specific terms to the general form of geometric sequences. Written responses frequently emphasized procedural progression from formula selection toward calculation steps rather than explicit mathematical justification.

Interview data further supported this pattern. When asked about the use of the formula, one student explained:

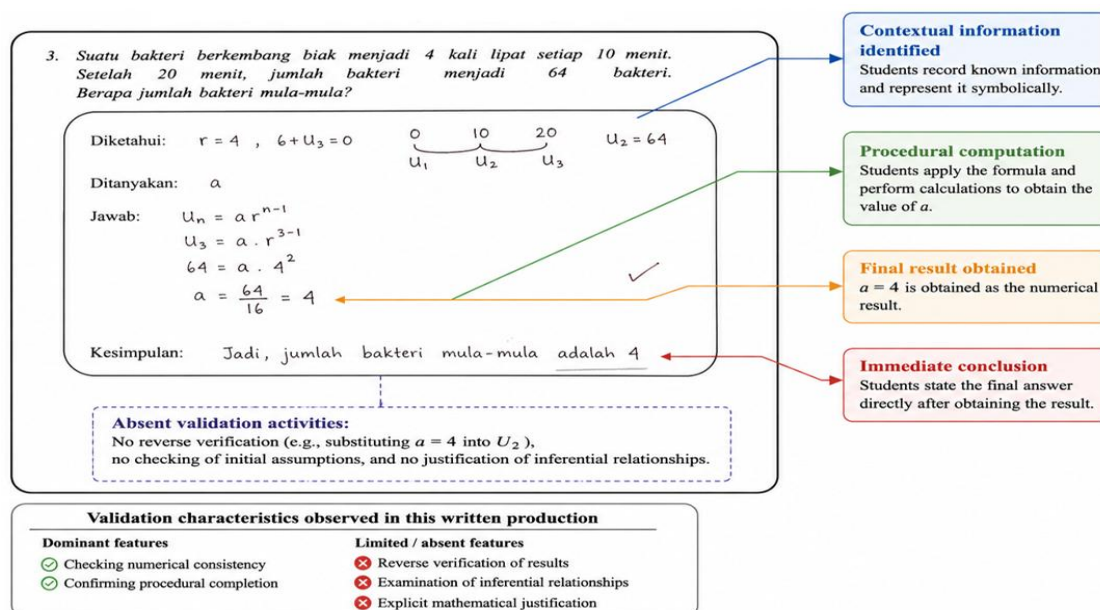
*Interviewer: How did your group determine the formula used in the solution?*

*Student G4: We remembered the geometric sequence formula and then substituted the values directly.*

The interview response suggests that students primarily relied on recalled formulas and procedural operations when communicating mathematical ideas. Similar patterns were identified across several groups, indicating that students frequently expressed their reasoning through symbolic procedures and formula-based representations, whereas conceptual explanations of relationships among quantities were less explicit in their mathematical arguments.

### Students' Mathematical Activities During the Validation Phase

The validation phase revealed important characteristics of students' proof-related activities, particularly regarding how mathematical correctness was established during group discussions. Classroom observations indicated that students generally evaluated the validity of their solutions by comparing numerical outcomes and examining procedural consistency rather than investigating the logical relationships underlying their arguments. During collaborative interactions, students often considered a solution acceptable when the final result matched expected values or when the same procedure had been used by other groups. Figure 3 presents a representative example of students' written production during the validation phase. The response illustrates how students completed the symbolic procedures and immediately proceeded toward the conclusion after obtaining numerical results.



**Figure 3.** Students' written production illustrating result-oriented validation during the validation phase

The written production indicates that students reached a numerical result and directly formulated a concluding statement without explicitly revisiting previous assumptions or verifying whether inferential relationships had been mathematically justified. The solution process ended once a computational result was obtained, suggesting that correctness was primarily associated with successful completion of the procedure. Although students correctly identified the first term and common ratio, their written work did not demonstrate reverse verification, such as substituting the obtained values into the previous relationships. For example, after determining:

$$a = 6$$

students did not further establish whether:

$$\begin{aligned} U_5 &= 6(2)^4 \\ &= 96 \end{aligned}$$

was satisfied through explicit verification procedures. Instead, students proceeded directly to the final statement of the solution. Interview data further supported this pattern. When students were asked whether they checked the correctness of their solutions, one participant explained:

*Interviewer: How did your group know that the answer was correct?*

*Student G3: Because the calculation was finished and the result matched what we expected.*

Another student similarly stated:

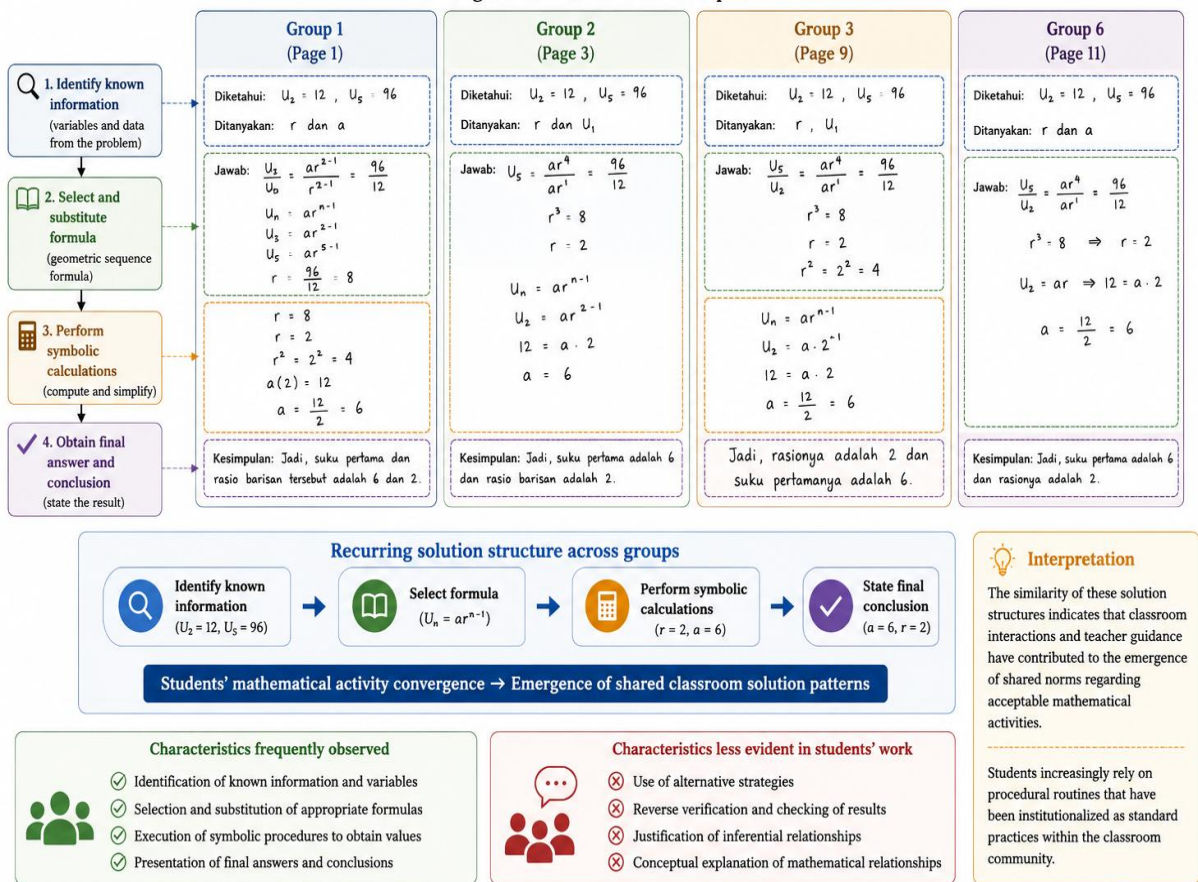
*Interviewer: Did your group check the answer again after obtaining the result?*

*Student G5: No, because we thought the formula had already been used correctly.*

These responses suggest that students predominantly relied on procedural completion and numerical agreement as criteria for validating mathematical solutions. Across several groups, similar patterns were observed in both written productions and classroom discussions. Students frequently evaluated mathematical solutions by symbolic consistency and peer agreement rather than by examining logical structures or inferential relationships. Validation activities therefore appeared to emphasize confirmation of outcomes rather than explicit construction of mathematical arguments.

### **Students' Mathematical Activities During the Institutionalization Phase**

During the institutionalization phase, students' mathematical ideas gradually became formalized through classroom discussions and teacher guidance. Classroom observations indicated that students increasingly expressed their solutions using similar mathematical structures and procedures. Across groups, solution processes began to exhibit comparable sequences of actions, suggesting the emergence of shared patterns of mathematical activity in the classroom. Figure 4 presents representative written productions from several groups during the institutionalization phase. The figure illustrates recurring solution structures observed across students' responses.

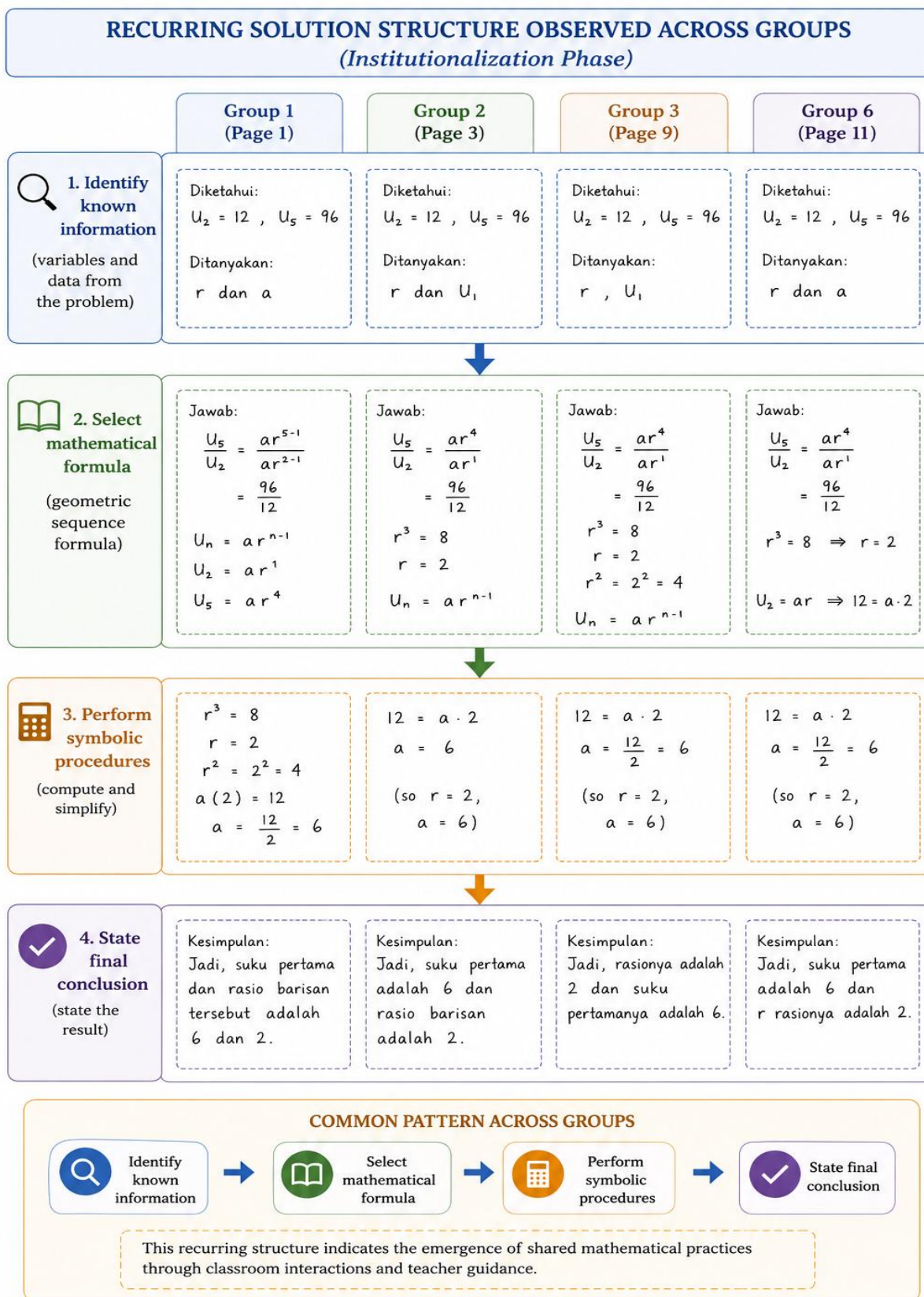


**Figure 4.** Students' written productions illustrating recurring mathematical structures during the institutionalization phase

The written productions reveal that students' mathematical responses increasingly converged toward similar organizational structures across groups. Despite differences in contextual situations and numerical values, students consistently employed comparable sequences of mathematical actions during problem solving. Rather than generating diverse approaches or alternative representations, students repeatedly adopted a common procedural pathway: identifying relevant information, selecting formulas, performing symbolic manipulations, and producing final conclusions.

A closer examination of the written responses indicates that the convergence among groups occurred not only in solution outcomes but also in procedural organization. Similar symbolic structures, formula selections, and sequences of operations recurred across written productions, suggesting that students increasingly organized their mathematical activities in line with shared classroom practices.

The recurring structure observed across groups can be summarized as follows:



**Figure 4.** Shared procedural structure emerging through classroom interactions during the institutionalization phase

Interview data further supported this pattern. When students were asked how their groups organized their solutions, one participant explained:

*Interviewer: Why did your group solve the problem in that way?*

*Student G2: Because that is usually the sequence we use when solving sequence problems.*

*Another student similarly stated:*

*Interviewer: Did your group consider using another approach?*

*Student G6: No, because we followed the method that had already been discussed in class.*

These responses indicate that students increasingly relied on commonly used procedures during classroom activities. Similar patterns appeared across multiple written productions and classroom discussions, suggesting that students' mathematical expressions gradually converged toward shared classroom practices.

### **Cross-Phase Synthesis of Students' Proof-Related Activities**

Across the four phases of the Theory of Didactical Situations, students' mathematical activities exhibited recurring characteristics in how they represented, communicated, and validated mathematical ideas. Evidence from written productions, classroom observations, and interview data showed that students repeatedly relied on symbolic procedures and formula-oriented approaches throughout problem-solving activities. Although the specific mathematical tasks differed, similar patterns of activity emerged across groups. To summarize the findings across phases, Table 1 presents the observed characteristics of students' mathematical activities within each TDS phase.

**Table 1.** Characteristics of Students' Mathematical Activities Across TDS Phases

<b>TDS Phase</b>	<b>Observed written activity</b>	<b>Interview/observation evidence</b>	<b>Emerging characteristics</b>
Action	Identification of known and unknown quantities; transformation into symbolic expressions	Students immediately identified the variables required for calculations	Initial symbolic orientation
Formulation	Formula-based representations and symbolic procedures	Students referred to remembered formulas during the discussion	Procedural communication
Validation	Checking numerical outcomes and consistency	Students regarded completed procedural calculations as evidence of correctness	Result-oriented validation



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Institutionalization	Repeated solution structures across groups	Similar sequences classroom interaction	problem-solving emerged during	Shared procedural pattern
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Overall, recurring patterns across phases indicate that students frequently approached mathematical problems through symbolic procedures and formula-based operations. Mathematical explanations tended to emphasize computational progression and the completion of procedures, whereas explicit articulation of inferential relationships and conceptual reasoning appeared less evident in written productions and classroom interactions.

## DISCUSSION

The findings demonstrate that students' mathematical activities across the phases of the Theory of Didactical Situations exhibited recurring tendencies toward symbolic and procedural engagement during problem solving. Although students generally produced mathematically correct solutions, evidence from written productions, classroom observations, and interview data suggests that mathematical activity was predominantly organized around procedural completion rather than explicit construction of inferential relationships. This finding indicates that successful completion of procedures does not necessarily correspond to meaningful mathematical understanding. Previous studies have similarly suggested that procedural and conceptual knowledge do not develop as independent constructs but interact dynamically during mathematical learning processes. However, procedural performance alone may not guarantee the emergence of conceptual relationships and mathematical justification (McCormick, 1997; Rittle-Johnson et al., 2015). This pattern is further reflected in students' tendency to rely on established procedures when approaching unfamiliar mathematical situations.

The action phase revealed that students initially engaged with mathematical tasks by identifying numerical information and immediately transforming contextual information into symbolic representations. Rather than exploring mathematical relationships embedded in the context, students focused primarily on identifying the variables and quantities required for calculations. Such findings suggest that students approached the problem situation through an operational accessibility lens, directing attention toward identifying mathematical elements perceived as useful for procedural completion. This finding is consistent with previous studies indicating that students frequently prioritize symbolic representations because these forms are perceived as more directly applicable for problem-solving activities (Novitasari et al., 2021). Mathematical representations function not merely as external expressions of thought but also as mediating structures through which mathematical ideas are organized and communicated (Goldin, 2020; Gérard, 1998). The immediate transformation of contextual



information into symbolic forms, therefore, reflects not only students' representational choices but also their initial orientation toward mathematical activity.

The formulation phase further showed that students increasingly relied on formula-based communication and symbolic procedures to express mathematical ideas. Although students successfully employed formal expressions such as geometric sequence formulas, explanations accompanying these representations remained comparatively procedural. Mathematical communication frequently involved recalling formulas and describing computational steps rather than explaining relationships among mathematical quantities. Previous studies have similarly shown that students may successfully produce mathematical expressions without explicitly articulating underlying reasoning processes (Cai et al., 1996). Consequently, the appearance of formal mathematical representations should not automatically be interpreted as evidence of conceptual understanding. Instead, symbolic expressions may function primarily as procedural tools supporting problem completion.

The validation phase revealed particularly important characteristics concerning proof-related activities. Students predominantly established correctness through numerical agreement and successful completion of procedures rather than through examination of inferential relationships and logical consistency. Students frequently considered solutions correct when calculations had been completed successfully or when results corresponded to expected outcomes. Similar findings have been reported in studies on proof validation showing that students often rely on procedural familiarity and outcome consistency as indicators of mathematical correctness (Komatsu et al., 2017). Moreover, validation processes that primarily focus on confirming results may limit opportunities for students to engage in broader forms of mathematical argumentation and proof construction (Powers et al., 2010). From a mathematical perspective, proof activity extends beyond verifying final answers to constructing relationships that explain why mathematical claims are valid (Richard et al., 2016). The present findings, therefore, suggest that students' validation activities remained predominantly result-oriented.

The institutionalization phase further demonstrated that students' mathematical activities gradually converged toward shared structures of problem solving. Across groups, students repeatedly adopted similar sequences involving the identification of information, the selection of formulas, symbolic procedures, and final conclusions. Such recurring structures indicate the emergence of common classroom practices guiding mathematical activity. Previous studies on sociomathematical norms suggest that classroom interactions progressively establish shared expectations regarding acceptable mathematical practices and forms of reasoning (Yackel et al., 2000). Consequently, mathematical activity may increasingly reflect collectively established norms rather than purely individual cognitive processes. Similar findings indicate that expectations within classroom environments may influence how students participate in mathematical activities and how mathematical ideas become socially legitimized (Planas & Gorgorió, 2004).



Collectively, these findings suggest that proof-related activity in school mathematics should not be understood solely as an individual cognitive phenomenon. Instead, students' mathematical arguments emerged through interactions among symbolic representations, classroom practices, and characteristics of the didactical milieu. This interpretation is consistent with didactical perspectives emphasizing that mathematical knowledge develops through interactions occurring within structured learning situations (González-Martín et al., 2014). Furthermore, learning environments may function as contexts that shape opportunities for students to engage in mathematical reasoning and proof-related activities (Laborde, 2000). The integration of perspectives on mathematical proof with the Theory of Didactical Situations, therefore, extends existing understandings of proof activity by showing that students' validation processes emerge not only from internal reasoning but also from interactions within the didactical environment. From an educational perspective, these findings suggest that instructional designs should create learning opportunities that encourage students not only to perform symbolic procedures but also to articulate inferential relationships and to explicitly justify mathematical decisions. Learning situations that emphasize explanation, argumentation, and reflective validation may offer stronger opportunities to develop richer forms of mathematical reasoning and proof construction.

## CONCLUSION AND SUGGESTION

### Conclusion

This study investigated how school students constructed and validated mathematical reasoning in learning geometric sequences through the lenses of the Theory of Didactical Situations and perspectives on mathematical proof. The findings revealed that students' mathematical activities across the phases of action, formulation, validation, and institutionalization demonstrated recurring procedural characteristics. During the action phase, students primarily initiated problem solving by identifying numerical information and immediately transforming contextual situations into symbolic expressions. During the formulation phase, students increasingly relied on formula-based representations and procedural communication to express mathematical ideas. The validation phase showed that students predominantly established correctness through numerical agreement and procedural completion rather than through explicit examination of inferential relationships. Furthermore, the institutionalization phase revealed the emergence of shared procedural structures across groups, suggesting the development of common mathematical practices within classroom interactions. These findings indicate that proof-related activities in school mathematics emerged not solely from individual cognitive processes but through interactions among students, symbolic representations, and the didactical milieu. Although students frequently generated mathematically correct solutions, explicit justification, inferential explanation, and conceptual articulation remained comparatively limited. The integration of mathematical proof perspectives with the Theory of Didactical Situations therefore contributes to a



broader understanding of mathematical activity by demonstrating that proof construction and validation processes are shaped not only by students' reasoning but also by the didactical conditions under which mathematical activity occurs.

### Suggestion

The findings suggest that mathematics instruction should provide more opportunities for students to engage in activities that require explicit justification and explanation of mathematical relationships, rather than focusing predominantly on procedural completion. Learning situations should encourage students to articulate inferential connections, compare alternative arguments, and reflect on the validity of mathematical conclusions. Such learning environments may support the development of richer forms of mathematical reasoning and proof activity. For future research, investigations may extend the present study to additional mathematical topics and educational levels to examine whether similar patterns of proof-related activities emerge across contexts. Future studies may also explore how modifications in the didactical milieu influence students' construction, validation, and communication of mathematical arguments.

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