



DEVELOPMENT OF STUDENT WORKSHEETS WITH THE HELP OF WORKED EXAMPLES ON MATRIX MATERIAL IN SENIOR HIGH SCHOOL

Firnanda Pradana Putra^{1*}, Ahmad Maulana Syafi'i², Anisa Dwiyana³, Putri Patricia⁴
Department of Mathematics Education, UIN Sultan Aji Muhammad Idris Samarinda, Indonesia;
pradana.0712@gmail.com , maulanasyafii95@gmail.com
anisadwiyana23073@gmail.com , patriciaputri2885@gmail.com

*Corresponding Author**

Firnanda Pradana Putra
UIN Sultan Aji Muhammad Idris Samarinda
pradana.0712@gmail.com
Contact Person: 0812-5349-5315

Article Info:

Received 2026-02-18
Revised 2026-05-04
Accepted 2026-05-17

How to Cite:

Putra, Firnanda Pradana., Syafi'i, Ahmad Maulana., Dwiyana, Anisa., Patricia, Putri (2026). Development of Student Worksheets with the Help of Worked Examples on Matrix Material in Senior High School. *Jurnal Theorems (The Original Research of Mathematics)*, 11(2), 100-111.

ABSTRACT

Mathematics is an important branch of science to learn in order to solve problems and present problems from contextual situations or conditions. This research includes research and development using the ADDIE model approach. This research produced a learning product in the form of worked examples that are suitable for use in mathematics learning on matrix material. The steps taken in this development were analysis, design, and development. The needs analysis was conducted by analyzing the matrix material found in phase E at the vocational high school/vocational high school level. The learning outcomes used were performing addition and subtraction operations on matrix forms. The design of the worksheet with the worked example approach included a template consisting of objectives, strategies, and concise material to help students learn matrices. In addition, providing story questions and clear steps for solving them can make it easier for students to understand the concept of matrices. Development is the final stage in research and development, which includes product design, feasibility testing, and product testing. The conclusion that can be drawn at this stage is that the learning product in the form of worked examples produces a product category that is suitable for use. The results from the validators, who are media experts and subject matter experts, show that the product is suitable for use without revision. The validators consisted of two education practitioners, designated as validator 1 and validator 2. The score obtained from validator 1 was 43 and validator 2 was 45, with an average of 44. Both validators produced a percentage of 88%, which is classified as good and requires no revision.

Keywords: Development, Worked Examples, Matrix, Senior High School.

INTRODUCTION

Learning is important for every individual in order to change their views and behavior when facing situations or conditions in life. According to Putra & Toni (2024), mathematics education is essential for expanding one's understanding and knowledge in everyday life. Learning can be defined as the process of changing an individual's abilities or behavior, who may not have had the skills before, to become more skilled through experience, practice, or interaction with the learning environment. According to Schunk (2016), learning is a lasting change in behavior or ability that is achieved in certain ways based on the results of practice or experience possessed by each individual. Learning requires thinking activities to acquire knowledge that can be used to solve problems encountered. According to



Kurniawati (2021), learning is a process that takes place within individuals, resulting in behavioral changes through thinking, attitude, and action. The learning process can involve and modify one's knowledge, skills, strategies, beliefs, attitudes, and behaviors, thereby gaining benefits that can be used in everyday life. This is consistent with Gagné's view that learning outcomes include verbal information, intellectual skills, cognitive strategies, attitudes, and motor skills (Gagne, et.al, 2019). Therefore, learning is an effort by each individual to acquire knowledge or skills, resulting in behavioral changes in dealing with situations or conditions in everyday life.

Mathematics is a branch of exact science that is important for every individual to learn in their daily lives. Students must study mathematics because it can shape critical, logical, analytical, creative, and systematic characters, as well as provide knowledge to deal with everyday problems. According to Sweller et al. (2019), mathematics is an important subject to be taught and learned by students in school because it can shape character and train abilities to support them in facing life's problems. Students can use mathematics in solving life problems that require critical and creative thinking skills, so that the solutions produced are accurate and appropriate. According to Chambers (2017), mathematics is an important science that serves as a tool for solving problems and discovering patterns, thereby training students' mathematical abilities to deal with real-life problems. Therefore, mathematics is a science that students need to master because it provides them with knowledge and shapes their mathematical character in dealing with everyday problems.

Learning mathematics involves mathematical understanding, which requires efforts to connect existing knowledge to the concepts being studied. According to Renkl et al., (2017), mathematical understanding can be formed when students are able to build new knowledge structures by connecting their existing knowledge with the knowledge being studied in the learning process. Students who have knowledge and experience in solving problems tend to find it easier to determine strategies and solutions to problems. However, students who are limited to their initial knowledge will find it difficult to deal with new situations or conditions, making it more difficult for them to solve problems. Therefore, learning mathematics involves mathematical problems that require problem solving skills supported by knowledge and experience in applying appropriate heuristic strategies.

Efforts to help students understand mathematical concepts can be aided by student worksheets. Student worksheets, or LKS for short, contain step by step strategies for problem solving. LKS can be compiled using a work example approach that contains concise material and mathematical examples that make it easier for students to understand mathematical concepts, as well as motivational sentences that encourage students to develop an interest in learning mathematics. In this case, students who are considered to have a high cognitive load will feel helped because they have steps for solving problems that make it easier to learn mathematics. According to Irwansyah & Retnowati, (2019), worked examples are a strategy in learning to solve problems that is considered very effective in reducing or

minimizing the cognitive load of students in learning mathematics. This is in line with the opinion of Clark et al. (2016), that worked examples are a strategy designed to reduce students' cognitive load caused by various forms of cognitive load resulting from the problem-solving process in learning. Worked examples also help students understand basic mathematical concepts, making them a useful strategy for teachers in mathematics learning. According to Sholikhah & Fahmi (2022), worked examples are part of cognitive load theory, which is a strategy used to improve learning effectiveness through problem solving. Worked examples can be used as a method of repetition in understanding mathematical concepts in learning. According to Nuraeni et al. (2023), worked examples not only present examples accompanied by solutions to problems, but also provide questions or problems of the same level of difficulty. This worksheet can also be used by students with different abilities. Therefore, worked examples are worksheets that students can use in mathematics learning to practice problem-solving skills and understand mathematical concepts, as well as reduce the cognitive load on students that is considered excessive or high.

Matrix are a mathematics subject for 10th grade or phase E at the senior high school level that requires an understanding of concepts in order to solve problems. Matrices involve rows and columns that can be used to present contextual problem information. However, students still experience difficulties in learning mathematics in the matrix subject. According to (Oroh et al., 2023), grade XI high school students still experience difficulties in learning matrices in the form of factual, conceptual, principled, and operational errors. This is explained in detail, namely conceptual errors in the form of errors in understanding abstract ideas; principle errors, namely errors in applying the principles in the questions; factual errors, namely errors related to the material in the questions; and operational errors, namely errors in performing operations on matrix addition in rows and columns. Therefore, an understanding of the basic concepts of matrices needs to be instilled in students so that they are able to solve the mathematical problems they encounter. Efforts to teach the concept of matrices can be made by providing worked examples that give sample questions and step by step solutions, as well as motivating students to learn. According to Deci & Ryan (2017), intrinsic motivation is crucial to the learning process. By motivating students and demonstrating the relevance of the material such as how matrix concepts are applied in everyday life they tend to engage more actively in learning. Based on the above discussion, students need to be guided and motivated in their study of mathematics so that they have the inner drive to continue developing their mathematical skills.

METHODS

This research is classified as development research because it produces educational products used in mathematics learning in the form of student worksheets with a worked example approach. There are various types of development models that can be used in development research, one of which is the

ADDIE model. According to Sari & Prasetyo (2020), the ADDIE model is a model that can be adapted to various development situations very well, so that it can still be used today. The ADDIE model consists of several stages, namely analysis, design, development, implementation, and evaluation. However, the development research conducted focused only on the development stage. According to Retnowati & Fadlila (2023), the steps that can be used to design learning products in the form of worked examples are analysis, design, and development. In general, the ADDIE model has the following cycle:

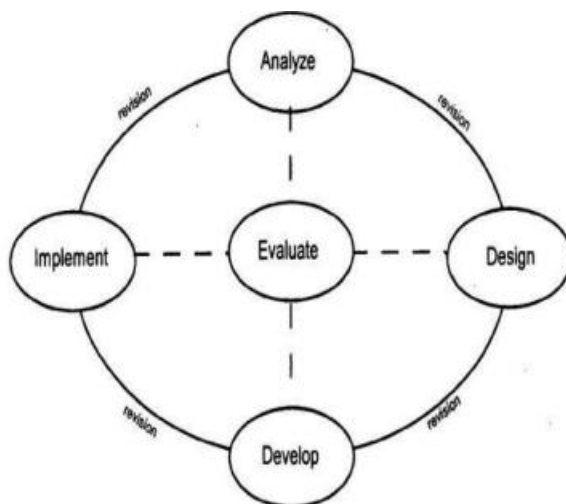


Figure 1. ADDIE Model Cycle.

In Figure 1, the ADDIE model was simplified according to the needs of the development research as follows:



Figure 2. ADD Model

The ADD model is a development model consisting of analysis, design, and development stages (Retnowati & Fadlila, 2023). The analysis stages carried out include student needs analysis, basic competency analysis (curriculum), and analysis of learning materials and determining the material. The design stage involves compiling worked examples, validation, and product evaluation. The development stage involves producing learning products in the form of worked examples that can be used in senior high school mathematics learning.

Why Stop at ADD?, because: 1) Focus on design and material development: By using only the ADD model, the focus is on the planning and creation of instructional materials specifically, developing worked examples tailored to students' needs. The ADD model allows for the creation of efficient instructional materials, even without immediately testing them or evaluating their effectiveness in the learning process, 2) Emphasis on Developing Necessary Materials: In many material development

studies, the Implementation and Evaluation stages can be conducted separately or after the development stage, involving pilot testing and evaluation in further research. By stopping at the ADD stage, the focus remains on creating and organizing the materials to be used in the learning process, and 3) Research Flexibility: The ADD model provides flexibility to develop materials that can later be tested in more in depth research during the Implementation and Evaluation stages (Gagne, et.al., 2019). This is very useful for producing sample problems that are solved as ready to use learning materials.

In the early stages of development, analysis was conducted by considering the needs of students who would use the worked examples. Grade 11 senior high school students were considered to still have difficulty solving matrix problems, so they needed worksheets with step by step solutions to understand the concept of matrices. In addition, an analysis of the curriculum was also conducted to determine the learning outcomes for matrices. In this case, students in phase E were considered to have already received the matrix material. The learning outcomes used were performing algebraic operations on matrices. The next stage was product or design development. The worked examples were developed by providing clear examples of problems and solutions that were not confusing, so that they were easy for students to understand. The worked example sheets were also designed to include motivational sentences to encourage students to persevere in facing problems. In addition, the worksheets were designed to provide relevant material to help students express the ideas used in problem solving. Before validation, the developers rechecked components such as language, tables, object placement, letters and numbers, colors, and the layout used in the worked examples. The final stage was the development of worked examples that are ready to be used in mathematics learning in the E phase of senior high school.

The data analysis technique for the feasibility of learning products is in the form of worked examples obtained from the validation assessment of media experts and subject matter experts. The scores given on the assessment sheet are as follows.

$$P = \frac{\sum x_i}{\sum X} \times 100\%$$

Description:

P = percentage

$\sum x_i$ = total score from validators

$\sum X$ = ideal total score.

Table 1. Eligibility Criteria

Percentage	Qualification	Criteria
$89 < score \leq 100$	Very good	No revisions
$74 < score \leq 89$	Good	No revisions
$64 < score \leq 74$	Fair	Revisions needed
$54 < score \leq 64$	Poor	Revisions
$0 < score \leq 54$	Very poor	Revisions

Sumber: Lestari, et. al., 2014 in (Lubis & Wahyuni, 2022)

FINDINGS

The following are the learning outcomes in the form of worked examples on matrix material developed using stages such as analysis, design, and development:

1. Analysis

The target users of this worked example-based worksheet are 10th grade high school/vocational school students in phase E. According to Sweller (2020), worked examples are a recommended learning design for students at the initial level. According to Ari Setiyani et al. (2025), students in phase E have been introduced to matrices to present and solve problems involving many variables or relationships in a concise and organized manner. At this phase, students can understand matrices to store and organize information, such as systems of equations, transformations, or connections in networks. In addition, students can use matrix operations as preparation or a foundation for further study in linear algebra or related fields.

2. Design

At the design stage, the worksheet based on worked examples contains learning objectives, learning strategies, and the time required to complete the problems. In addition, this sheet also provides summary material that makes it easier for students to connect the information encountered in the problems with their existing knowledge. This worksheet is designed using colors to create a positive impression for students in mathematics learning. According to Hagan (2020), aesthetic elements, including color, can boost students' motivation and mood, create a more enjoyable learning experience, and increase their engagement. The appropriate use of color can improve students' perception of the subject matter and help them stay focused and feel more positive about the learning process.

The design of this worksheet also considers students who do not yet fully understand the concept of matrices. For students who have a good understanding of matrix material, this worksheet can be used to broaden their knowledge with different types of questions. Vygotsky explains that the most effective learning occurs when students learn within their zone of proximal development (ZPD) the area between what they can already do on their own and what they can do with assistance or guidance. In this context, worksheets that focus on more challenging problems can foster further development in students who have mastered the foundational material by providing them with challenges appropriate to their skill level.

The following is Figure 3 related to the worksheet on matrix material that has been compiled.

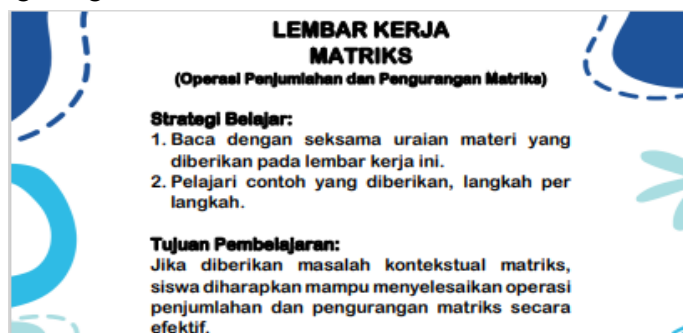


Figure 3. Homepage Worked Examples

It is necessary to remind students of all the information they have learned. This worksheet with sample problems is designed to help students review the material they have previously studied. With this summary, students can review key concepts related to matrices, thereby reinforcing their understanding. Additionally, this worksheet presents solved example problems along with detailed solution steps, aimed at demonstrating the application of these concepts in a more practical context. Through this process, students not only understand the basic theory underlying matrix concepts but also learn how to apply their knowledge to solve more complex problems, which ultimately helps them understand and master the topic.. Figure 4 below is an example of a matrix question and its solution.

Belajar adalah Kunci untuk Membuka Pintu Kesuksesan. Ayo pelajari Matriks di bawah ini agar Kamu terus Tumbuh dan Berkembang.

(Waktu: 7 Menit)

1. Toko buku "Cerdas" mencatat penjualan buku selama dua hari:

Hari Pertama:

Penjualan buku	Matematika	IPA
Toko A	12	8
Toko B	9	6

Hari Kedua:

Penjualan buku	Matematika	IPA
Toko A	9	6
Toko B	11	11

Ayo cermati contoh berikut dan perhatikan langkah per langkah... semangat, kamu pasti bisa!

a. Nyatakan data pada tabel di atas dalam bentuk matriks dan berilah notasi pada matriks tersebut!
b. Berapakah banyak buku Matematika dan IPA yang dijual oleh Toko A dan B selama 2 hari?

Penyelesaian:

Diketahui: Penjualan Toko Buku "Cerdas" yang Disajikan dalam Bentuk Matriks:

Figure 4. Example Questions and Solution Steps


In Figure 4 above, motivational statements are intended to encourage students not to be afraid to understand the step-by-step process of solving matrix operations and additions. According to Slavin (2024), motivational statements are an effort to raise enthusiasm and encourage students to actively participate in learning. In addition, motivational sentences such as “you can do it” and “keep up the good work” are expected to reduce students' anxiety in learning mathematics. Similarly, Ashcraft & Krause (2019) also stated that sentences containing positive affirmations can reduce anxiety and improve mathematics learning performance. In this case, giving positive sentences can encourage students to be more interested in completing tasks, so that they feel appreciated and challenged in a positive way. In the solution steps, worked examples are also provided with explanations such as “add the elements that are in the same place” so that students can easily understand the material and be able to solve problems. According to Rittle-Johnson & Schneider, (2024), providing explanations at each step of the solution can improve conceptual understanding and flexibility of strategies used by students in solving mathematical problems. In addition, providing explanations can reduce the cognitive load on students, which is considered quite heavy in understanding mathematics. According to Sweller et al. (2019), worked examples accompanied by explanations at each step can reduce cognitive load and increase learning efficiency, thereby reducing students' learning anxiety. Therefore, worksheets with a worked example approach are designed with clear solution steps and motivational sentences so that students are able to learn mathematics on matrix material well.

Worksheets with worked examples are designed to include contextual problems related to everyday life, so that students can learn mathematics in a meaningful way. Contextual problems

can provide students with experiences that enable them to think critically and reason in solving problems. According to NCTM (2018), in a book entitled Principles and Standards for School Mathematics, it is recommended that contextual problems be used to develop students' mathematical abilities in terms of problem solving and reasoning. Contextual problems make abstract mathematical concepts more concrete, making it easier for students to build a meaningful understanding of mathematics. In addition, the presentation of problems on the worksheets also pays attention to the use of appropriate and clear fonts and letters, so that students are able to read and are interested in understanding the questions and learning independently.

3. Development

Based on the worksheet results with the help of the designed worked examples, students can read the learning strategies, objectives, time allocation, and summary material to increase their insight and knowledge about matrix material. Students can learn independently by reading carefully based on the instructions provided in the worksheet. According to Schonk (2017), self reflection is an important part of the learning process. Students who actively evaluate their progress for example, by completing questionnaires about their feelings regarding their learning can improve their understanding of the material and reinforce their learning. At the end of the session, students are asked to fill out a questionnaire to determine their mood regarding their learning progress with the help of worked examples. The following is the questionnaire provided in the worksheet.

Coba kamu isi Tabel di bawah ini dengan yang dirasakan sekarang ya? 

Seberapa mudah atau sulitkah masalah tersebut diselesaikan?	1	2	3	4	5	6	7	8	9
	Sangat Mudah Sekali								Sangat Sulit Sekali

Pertanyaan	0%	10%	20%	30%	50%	60%	70%	80%	90%	100%
	Sangat Tidak Yakin									Sangat Yakin
Mengubah data tabel menjadi notasi matriks										
Menyelesaikan operasi penjumlahan matriks										
Menyelesaikan operasi pengurangan matriks										
Menjawab masalah										

Figure 5. Questionnaire on Worksheets

Questionnaires after completing mathematics problems are non-cognitive evaluation tools used to determine students' reactions, perceptions, and attitudes toward the learning process or media used. The use of these questionnaires has important benefits in learning and teaching instrument development, including in the context of research and development (R&D). In development research, post-activity questionnaires are used to find out whether the products developed are effective, interesting, and easy to use according to



students. Borg & Gall (2018) explain that one of the important instruments in the formative and summative evaluation of development products is student response questionnaires. Therefore, student input in the development of worked examples is important for improving learning products.

Development is the final stage in research and development, which includes product design, feasibility testing, and product trials. The conclusion that can be drawn at this stage is that learning products in the form of worked examples produce a category of products that are suitable for use. The results from the validators, who are media experts and subject matter experts, indicate that the products are suitable for use with revisions. The validators consisted of two education practitioners, assigned as validator 1 and validator 2. The score obtained from validator 1 was 43 and validator 2 was 45, with an average of 44. Both validators produced a percentage of 88%, which is classified as good and requires no revision.

CONCLUSION AND SUGGESTION

Based on the results and discussion above, the development of worked example learning products on matrix material has led to the following conclusions and recommendations.

Worksheets with worked examples are designed for senior high school students in phase E who are just beginning to learn about matrices, particularly addition and subtraction operations. Using a worked example approach, which is recommended for beginners, these worksheets present sample questions and their solutions to help students understand the concepts and solve contextual problems more easily and purposefully. Motivational sentences in mathematics worksheets are used to foster enthusiasm for learning, reduce anxiety, and increase students' emotional engagement. Affirmative sentences such as "You Can Do It" provide psychological support that encourages students to actively understand the steps to solve problems. In addition, the inclusion of explanations in each step of the solution, such as technical instructions in the worked examples, helps improve conceptual understanding and reduce students' cognitive load. The use of contextual questions also provides tangible benefits in relating the material to everyday life, which can improve students' critical thinking and problem-solving skills. Visually, the use of clear fonts and letters also plays a role in attracting students' attention and facilitating independent learning.

ACKNOWLEDGMENTS

Thank you for the support from various parties, which has enabled this article to be completed successfully. Thank you to the mathematics education study program at Sultan Aji Muhammad Idris



State Islamic University Samarinda for encouraging the improvement of scientific writing in the form of articles.

REFERENCE

- Ari Setiyani, Yudi Kustiana, Sri Wantini, Setiawan Agung Wibowo, Kiki Ariyanti Sugeng, & Dicky Susanto. (2025). *Panduan mata pelajaran matematika fase a - f dan fase f tingkat lanjut*. Badan Standar, Kurikulum, dan Asesmen Pendidikan Kementerian Pendidikan Dasar dan Menengah Republik Indonesia.
- Ashcraft, Mark H., & Krause, Justin A. (2019). Working memory, math performance, and math anxiety: A review of recent findings. *Journal of Applied Research in Memory and Cognition*, 8(1), 1-12. <https://doi.org/10.1016/j.jarmac.2018.10.004> .
- Borg, Walter R., & Gall, Meredith D. (2018). *Educational research: An introduction* (10th ed.). Pearson.
- Chambers, P. (2017). *Teaching mathematics: Developing as a reflective secondary teacher* (2nd ed.). Sage.
- Clark, Richard C., Nguyen, Fred, & Sweller, John. (2016). *Efficiency in learning: Evidence-based guidelines to manage cognitive load* (2nd ed.). John Wiley & Sons.
- Deci, Edward L., & Ryan, Richard M. (2017). *Self-determination theory: 35 years of progress and future directions*. In *Advances in motivation and achievement* (Vol. 18, pp. 127–166). Emerald Publishing Limited. <https://doi.org/10.1108/S1876-336320170000018008>
- Gagné, Robert M., Wager, Walter W., Golas, Kathryn C., & Keller, John M. (2019). *Principles of instructional design* (6th ed.). Cengage Learning.
- Hagan, L. M. (2020). *The role of aesthetic learning in fostering mathematical creativity and engagement*. *Educational Studies in Mathematics*, 104(1), 1-16. <https://doi.org/10.1007/s10649-020-09924-5>.
- Sari, I. P., & Prasetyo, E. (2020). Pengembangan bahan ajar matematika berbasis model ADDIE pada materi operasi hitung bilangan bulat. *Jurnal Pendidikan Matematika*, 14(1), 54-67. <https://doi.org/10.1234/jpm.v14i1.2020>
- Schön, Donald A. (2017). *The reflective practitioner: How professionals think in action* (2nd ed.). Routledge.
- Schunk, D. H. (2016). *Learning theories: An educational perspective* (7th ed.). Pearson.
- Irwansyah, M. F., & Retnowati, E. (2019). Efektivitas *worked examples* dengan strategi pengelompokan siswa ditinjau dari kemampuan pemecahan masalah dan cognitive load. *Jurnal Riset Pendidikan Matematika*, 6(1), 62–74.
- Kurniawati, W. (2021). Desain perencanaan pembelajaran. *Jurnal An-Nur: Kajian Ilmu-Ilmu Pendidikan Dan Keislaman*, 7(01), 1–10.
- Lubis, R. A., & Wahyuni, S. (2022). Development of mathematics student worksheets based on mathematical understanding with a worked example approach. *EduMatika: Jurnal MIPA*, 2(3), 69–74.
- NCTM. (2018). Principles and standards for school mathematics. *Council of Teachers of Mathematics*.



- Nuraeni, Z., Simarmata, R. H., & Tarigan, A. H. Z. (2023). Implementation worked example-based learning to improve junior high school students' mathematical representation ability. *Jurnal Pendidikan Matematika (JUPITEK)*, 6(1), 36–41.
- Oroh, V., Manurung, O., & Tumulun, N. K. (2023). Analisis kesalahan peserta didik dalam menyelesaikan soal matematika materi operasi matriks. *Jurnal Pendidikan Dan Keguruan*, 1(2), 732–741.
- Putra, F. P., & Toni, T. (2024). Analysis of the Algebraic Reasoning Ability of State Madrasah Tsanawiyah Students in Solving Mathematical Problems based on Cognitive Style. *JTMT: Journal Tadris Matematika*, 5(2), 128-137.
- Renkl, Andreas, Atkinson, Richard K., Maier, Udo H., & Staley, Richard. (2017). From example study to problem solving: Smooth transitions help learning. *Educational Psychology Review*, 29(4), 749–776. <https://doi.org/10.1007/s10648-017-9397-0>.
- Retnowati, E., & Fadlila, N. (2023). The compound area of quadrilaterals and triangles: a worked example based learning design. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, 7(1), 150.
- Rittle-Johnson, Bethany, & Schneider, Markus. (2020). Developing conceptual and procedural knowledge of mathematics: A systematic review of research. *Educational Psychology Review*, 32(4), 829-868. <https://doi.org/10.1007/s10648-020-09500-w>.
- Sholikhah, S., & Fahmi, S. (2022). Desain pembelajaran worked example pada materi trigonometri. *Seminar Nasional Hasil Pelaksanaan Program Pengenalan Lapangan Persekolahan*, 1471–1480.
- Slavin, Robert E. (2024). *Educational psychology: Theory and practice* (14th ed.). Pearson.
- Sweller, J. (2020). Cognitive load theory and educational technology. *Educational Technology Research and Development*, 68(1), 1–16.
- Sweller, John, Ayres, Paul, & Kalyuga, Slava. (2019). *Cognitive load theory* (2nd ed.). Springer.
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.