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APPLICATION OF CONGRUENCE FROM NUMBER THEORY TO NIM GAMES

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ABSTRACT

Not only are numbers, formulas and symbols common in maths, but there are also games that can be learnt as an addition to the skills that refer to critical thinking. In mathematics, the simple application of games developed in number theory is a form of reflection related to mathematics. One of the applications of number theory that games are capable of is the NIM game. NIM game is an old traditional game that has a strategy in solving the game. In this game, which is carried out by two players, each player has his turn alternately to win it with the rules of this game. The success of winning the NIM game is that there are objects available and undergo the game with the concept (strategy) as a way to take the intended object. A player who takes the last object is the winner of the game. The following will describe 7 NIM games: NIM Maxima, NIM One-Four, NIM Three-Five, NIM One-Three-Four, NIM One-Three-Five, NIM One-Three-Five-Seven and NIM Two-Four-Nine-Eight. This NIM game is a form of application of mathematics in number theory that focuses on congruence theory as a way to determine the winning strategy in the NIM game.

Keywords: Application of Congruency, Strategy, NIM Game

INTRODUCTION

In general, games are a simple form of activity whose purpose is entertainment, leisure, or light exercise. However, with the development of the times it began to change. The changes are characterised by the emergence of various ideas that show that the game can undergo changes. (Hidayat, 2019). So it can be interpreted, that a game is something to play with certain rules so as to get a win or lose in the game that is carried out, which is solely as a refreshing. (Retno, 2015). Play is the same as learning, learning is an activity that is intentional and carried out by individuals so that changes in their abilities occur. (Muharam et al., 2023). There are aspects of development in play that consist of several aspects, namely: cognitive, language, psychomotor, and socio-emotional. In these aspects based on the type of

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game (Wahab & Rosnawati, 2021). The levels of the game can increase the development of the intended aspect depending on the challenges posed to the person playing it.

In mathematics, there are also games that can hone skills, namely psychomotor skills. This psychomotor ability is an ability with quick thinking skills in order to win a game. According to Siedentop, Herkowitz, and Rink, play is a physical activity that is done voluntarily, separated between its scope and breadth. (Utama, 2020). Menurut Huizinga manusia adalah homo ludens yaitu manusia atau makhluk hidup yang suka bermain (Nurcahyo, 2018). According to Vygotsky, play has a direct role in the development of cognition. (Khadijah & Armanila, 2021). So in maths, games can be used as a reflection and a dexterity enhancer. In this case, games can build an ability (Susanti, 2021) associated with learning will form simple ideas or notions in solving its challenges. The ability to solve a problem can be a challenge for someone who does it (Harefa et al., 2024). Please note that each game has different methods, strategies, missions, and ways to complete it to achieve a victory.

NIM is an old (ancient) game with a certain strategy in achieving its victory. (Yong et al., 2016). Permainan ini bermula dari Negeri Bambu yang disebut "Tsyanshidzi" atau mengangkat. Another history also states that the name NIM originated from the German expression "Nimm" which means take or unplug. (Tehuayo et al., 2017). NIM is a game that has existed for a long time in antiquity. Then the expert who introduced the NIM game was in the 15th century. One of the Harvard mathematics professors, Charles Bouton, was the one who disseminated the concept of this NIM game. So this game is a game that has the concept (strategy) of victory played by two players. The mathematical theory that appears in the application of this NIM game is number theory in the application of congruence theory. NIM is able to hone dexterity in thinking quickly.

The purpose in the development of the application of congruence theory is to explain and apply an advantage of the concept of number theory to the application of congruence theory through NIM games. Then, to find out the differences in NIM games through literature review of previous journals. The first journal by Benny Yong, et al with the title "Application of Number Theory in NIM Games" in the journal only discusses four NIM games namely: NIM Maxima, NIM One-Four, NIM One-Three-Four, and NIM One-Three-Five-Seven. There are general characteristics of the game. Then, the second journal by Tehuayo, et al with the title "Application of Congruence Theory in NIM Games" in the journal discusses the NIM Game which consists of seven NIM games, namely: NIM Maxima, NIM One-Three, NIM One-Four, NIM One-Two-Four, NIM One-Three-Four, NIM One-Three-Five-Seven and NIM Three-Five-Seven-Nine. There are more explanations about things related to NIM games such as: Combinatorial Games, definitions and theorems underlying NIM games and a complete history of NIM games. Based on this development, the following seven applications of NIM games are focused on, namely: NIM Maxima, NIM One-Four, NIM Three-Five, NIM One-Three-Four, NIM One-Three-Five, NIM One-Three-Five-Seven, and NIM Two-Four-Nine-Eight.

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METHODS

. In writing this development research using the literature review method, by reviewing several journals related to the application of number theory through games and from number theory books. The following definitions and properties of congruence are the basis for the application of NIM games.

1.1. Definition and Properties of Congruence

a. Definition 2.1 (Sari, A., 2018)

Specified $\boldsymbol{a, b, m \in Z}$

$\boldsymbol{a}$ is called congruent modulo $\boldsymbol{b}$ $\boldsymbol{modulo}$ $\boldsymbol{m}$, written $\boldsymbol{a \equiv b\ (mod\ m)}$, if $\boldsymbol{a(a-b)}$ is disivible by m, i.e. $\boldsymbol{m \mid (a-b)}$.

If $\boldsymbol{(a-b)}$ is not divisible by $\boldsymbol{m}$, then it is written $\boldsymbol{a \equiv b\ (mod\ m)}$, read $\boldsymbol{a}$ is not congruent modulo $\boldsymbol{m}$.

When $\boldsymbol{(a-b)}$ is divisible by m ⇔$\boldsymbol{(a-b)}$ is divisible by $\boldsymbol{-m}$, then:

$$\boldsymbol{a \equiv b\ (mod\ m) \Leftrightarrow b \equiv a\ (mod\ (m))}$$

b. Theorem 2.1

If $\boldsymbol{a, b}$ and $\boldsymbol{m}$ is the set of integers where m is positive, such that $\boldsymbol{a \equiv b\ (mod\ m)}$ ⇔ there is an integer k such that $\boldsymbol{a = mk + b}$

c. Example Defnition 2.1

$\boldsymbol{8 \equiv 4\ (mod\ 2)}$ because 2 | (8-4) or 2 | 4

$\boldsymbol{5 \equiv -4\ (mod\ 9)}$ because 9 | {5- (-4)} or 9 | 9

$\boldsymbol{107 \equiv 2\ (mod\ 7)}$ because 7 | (107 – 2) or 7 | 105

In fact, congruence problems often arise in everyday life. Examples of its application in life are the work of watches that use modulo 12 to express hours, the use of modulo 6 to express minutes and seconds, the use of modulo 7 from calendar work for the days of the week, the use of modulo 5 for market days in Java (Pon, Wage, Kliwon, Legi, Pahing), and the use of modulo 12 for months in years.

1.2. Congruence in Number Theory

$\boldsymbol{a}$ declared congruent $\boldsymbol{b}$ modulo $\boldsymbol{m}$ $\boldsymbol{(a \equiv b\ mod\ m)}$, if $\boldsymbol{a}$ has a remainder equal to $\boldsymbol{b}$ when $\boldsymbol{a}$ and $\boldsymbol{b}$ both divided by $\boldsymbol{m}$. Example: $\boldsymbol{4 \equiv 0\ mod\ 2, 24 \equiv 17\ mod\ 7, dan\ 3 \equiv -5\ mod\ 8}$

FINDINGS

2.1. Strategy in the NIM Game

1. There are 2 players, as the first player and the second player (Arifin, n.d.)
2. When starting the game, place any number of objects (odd or even numbers) without counting them first. Particularly for NIM games that have 4 (four) NIM numbers, each object must be arranged in accordance with the predetermined number sequence.

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3. Then determine the type of NIM game to be played based on the agreement of the 2 players.
4. Each player has the opportunity to take turns playing.
5. After determining the NIM to be played, use the scheme (strategy) in accordance with the NIM rules that have been set.
6. When playing NIM, do it in accordance with the congruence theory in modulo that has been established.
7. In this game, the opponent's carelessness is the main key to victory in the NIM game.
8. There is no element of chance like a dice game

2.2. NIM Maksima Game Scheme

In the NIM Maksima game, there is a person who plays it has an object, which is symbolised by M objects and the number of objects taken for each turn of objects is symbolised by N, so that $M > N$; $(M, N \in \mathbb{Z}+)$ (Yong et al., 2016). There are 2 players (e.g. player A and player B) and M objects on the table. The players in their turn can only take at most 4 objects. The winning position of NIM Maxima with its strategy is that the player who takes the last remaining object is the winner. Here is an illustrative example of the NIM Maxima game with its winning strategy.

Proof I: (Player A starts first and is the winner)

Two players take turns playing and removing 1,2,3, or 4 objects from an array of 26 objects. The player who takes the last object is the winner. Here is the flow of the game that player A as the first player is able to win the game (*Permainan NIM*, 2019).

Game Flow:

The victory of this game is obtained by the backward strategy. The first player (player A) can win if in the last step he takes the remaining 1,2,3, or 4 objects. Then the second player (player B) must leave 1,2,3, or 4 objects, if there are 5 objects that must be removed from the arrangement of objects. Then player A in the first flow must leave 5 objects for player B, if there are 6,7,8 or 9 objects left, then player B must set aside objects from 10 arrays of objects. Then player A must leave 10 objects for player B in the second flow, if there are 11,12,13, or 14 objects left, then player B must set aside objects from the 15 object array. Then, player A must leave 15 objects for player B in the third flow, if there are 16,17,18, or 19 objects left, then player B must set aside objects from the 20 object array. Then player A must leave 20 objects for player B in the fourth flow, if there are 21,22,23, or 24 objects left, then player A must leave 25 objects.

Based on the illustration above, it shows that the first player (player A) will win the game, if it leaves 25,20,15,10, and 5 objects for the second player (player B). So player A can win if player A puts the remaining objects with:

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$$\mathbf{0}\ \boldsymbol{mod}\ \mathbf{5}\ \boldsymbol{or}\ \mathbf{0}\ \boldsymbol{mod}\ (\boldsymbol{n}+\mathbf{1}), \boldsymbol{Where}\ \boldsymbol{n}=\mathbf{4}$$

To understand the above evidence, consider table 1 below.

Table 1. Proof of NIM Maxima over 26 objects, victory condition that the player who takes the object at the end is the winner (Player A starts first and is the winner)

Player A		Player B	
Object Taken	**Remaining Objects**	**Object Taken**	**Remaining Objects**
1	25	4	21
1	20	3	17
2	15	2	13
3	10	2	8
3	5	2	3
3	0		

Proof II: (Player A starts first, player B is the winner)

Two players take turns playing and removing 1,2,3, or 4 objects from an array of 25 objects. The player who takes the last object is the winner. Here is the game flow that player A is the first player but player B wins the game.

Game Flow:

The victory of this game is obtained by backward strategy. Victory in the second player (player B) can be obtained if the last pick of the remaining objects is 1, 2, 3, or 4 objects then player A must leave 1, 2, 3, or 4 objects by setting aside objects from 5 arrays of objects. Then player B must set aside 5 objects for player A, leaving 6, 7, 8, or 9 objects to be removed from the 10-object array. Then, player B sets aside 11, 12, 13, or 14 objects and player A must remove 15 objects from the array. The same steps are performed as in illustration I.

Thus player B can achieve success if player B places the remaining objects with:

$$\mathbf{0}\ \boldsymbol{mod}\ \mathbf{5}\ \boldsymbol{or}\ \mathbf{0}\ \boldsymbol{mod}\ (\boldsymbol{n}+\mathbf{1}), \boldsymbol{Where}\ \boldsymbol{n}\ =\ \mathbf{4}$$

To understand the above evidence, consider the following table 2.

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p-ISSN: 2541-4321

Tabel 2. Proof of NIM Maxima game through 25 objects, the victory condition is that the player who takes the object at the end is the winner (Player A starts first, Player B is the winner).

Player A		Player B	
Objects Taken	**Remaining Objects**	**Objects Taken**	**Remaining Objects**
1	24	4	20
3	17	2	15
1	14	4	10
3	7	2	5
1	4	4	0

After applying Proof I and Proof II, it is found that the victory of NIM Maksiama is won by always leaving objects, namely$\boldsymbol{0\ mod\ 5\ or\ 0\ mod\ (n+1), Where\ n\ =\ 4}$

Based on the application of NIM Maxima, it can be seen that regardless of the number of cards (odd or even) that will be played, the victory by the player will be obtained by fulfilling the condition where the player with the last remaining object is the winner. For the next NIM victory, it will have the same method as the Maxima NIM game depending on the completion of the predetermined winning strategy.

2.3. NIM One-Four Game Scheme

Suppose player A and player B have M objects placed on the table and the number of times to pick up objects is 1 or 4 of the number of objects available. The game can achieve success by the player who is the last to bring/take the remaining objects.

Proof III: (Player A starts first, but player B is the winner)

Two players take turns playing and removing 1 or 4 objects from an array of 17 objects. The player who takes the last object is the winner. Here is the game flow that player A or player B can win the game.

Game Flow:

This game is won with a backward strategy. The second player (player B) can win if he takes the last remaining 1 or 4 objects. Player A must set aside 1 or 4 objects from 2 or 5 objects from the object array. Then the last move of player B must leave 2 or 5 objects to player B with 3,6 or 9 objects left while player A leaves objects from 7 or 10 object arrays Then player B must leave 7 or 10 objects to player A in the second flow with 8,11, or 14 objects left while player A leaves 12 or 15 objects from object arrays. Then player B must leave 12 or 15 objects to player A in the third flow with 16 objects left for player A.

e-ISSN: 2528-102X
p-ISSN: 2541-4321

Thus the proof shows that player B can win the game leaving 15, 12, 10, 7, 5, and 2 objects for player A. So player B wins the game by leaving as many objects:

$$\mathbf{0}\ \boldsymbol{mod}\ \mathbf{5}\ \boldsymbol{or}\ \mathbf{2}\ \boldsymbol{mod}\ \mathbf{5}$$

To understand the above evidence, consider table 3 below..

Table 3. Proof of the game NIM One-Four through 17 objects, the victory condition is that the player who takes the object at the end is the winner (Player A starts first, but player B is the winner).

Player A		Player B	
Objects Taken	Remaining Objects	Objects Taken	Remaining Objects
1	16	1	15
4	11	4	7
1	6	4	2
1	1	1	0

Setelah After applying Proof III, it is found that the victory of NIM One-Four is won by always leaving an object, namely $\mathbf{0}\ \boldsymbol{mod}\ \mathbf{5}\ \boldsymbol{or}\ \mathbf{2}\ \boldsymbol{mod}\ \mathbf{5}$

2.4. Three-Five NIM Game Scheme

Suppose player A and player B have M objects placed on the table and the number of times to take objects is 3 or 5 of the number of objects available. This game can be won by the last player who brings/takes the remaining objects.

Proof IV: (Player A starts first, but player B is the winner)

Two players take turns playing and removing 3 or 5 objects from a pile of 35 objects. The player who takes the last object is the winner. Here is the game flow that player A or player B can win the game.

Game flow:

The victory of this game is obtained by the backward strategy. The final flow in the second player (player B) can be won by taking the last remaining object of 3 or 5 objects and leaving objects less than 3. The first player (player A) must leave 3 or 5 objects by leaving 4 or 6 objects. Then the flow at the end of player A must leave 4 or 6 objects for player B with 6, 7, or 9 objects remaining and player B must leave 8 or 10 objects from the array of objects. Then player A removes 8 or 10 objects to player B in a second move with 9, 12, or 16 objects left while player B has 13 or 24 objects left from the array. Then player A must set aside 13 or 24 objects for player B in the third flow with player A having to leave 29 objects for player B.

e-ISSN: 2528-102X
p-ISSN: 2541-4321

Thus the above illustration shows that player B can win the game by leaving 32, 29, 24, 21, 16, 13, 10, 5, and 2 objects for player A. So player B can win the game by leaving as many objects:

$$\mathbf{0\ \mathit{mod}\ 8\ \mathit{or}\ 2\ \mathit{mod}\ 8}$$

To understand the above evidence, consider the following table 4..

Table 4. Evidence of the game NIM Three-Five through 35 with, the victory of the player who takes the object at the end is the winner or leaves less than 3 objects is the winning player (Player A starts first, but player B is the winner)

Player A		Player B	
Objects Taken	Remaining Objects	Objects Taken	Remaining Objects
3	32	3	29
5	24	3	21
5	16	3	13
3	10	5	5
3	2		

After applying Proof IV, it is found that the victory of NIM Three-Five is won by always leaving an object, namely $\mathbf{0\ \mathit{mod}\ 8\ \mathit{or}\ 2\ \mathit{mod}\ 8}$

2.5. NIM One-Three-Four Game Scheme

Suppose player A and player B have M objects placed on the table and the number of times to pick up objects is 1,3 or 4 of the number of objects available. The game can be won by the player who is the last to bring/take the remaining objects.

Proof V: (Player A starts first and is the winner)

Two players take turns playing and removing 1,3 or 4 objects from a 29 object array. The player who takes the last object is the winner. Here is the game flow that player A or player B can win the game.

Game Flow:

Winning this game is achieved with a backward strategy. The first player (player A) can win on the last step by taking the remaining 1,3 or 4 objects. Player B must leave 1,3 or 4 objects out of 2 or 7 objects from the array of objects. Then player A in the last flow must set aside 2 or 7 objects for player B with 5, 6, 9, 12 or 13 objects left, while player B has 9 or 14 objects left from the array. Then player A must set aside 9 or 14 objects for player B on the second flow with 12, 13, 15, 17, 18, or 20 objects remaining when player B has 16 or 21 objects left from the array. Then player A must set aside 16 or 21 objects for player B in the third flow with 19, 20, 22, 24, 25, or 27 objects remaining when player A must leave 23 or 28 objects from the array. Thus, the above

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illustration shows that player A can win the game by leaving 28,23,21,16,14,9,7, and 2 objects for player B, respectively. So player A can win the game by leaving as many objects:

$$\mathbf{0\ mod\ 7\ or\ 2\ mod\ 7}$$

In order to understand the above evidence, consider the following table 5:

Table 5. Evidence of the NIM One-Three-Four game through 29 objects, the victory condition of the player who takes the object at the end is the winner (Player A starts first and is the winner).

Player A		Player B	
Objects Taken	Remaining Objects	Objects Taken	Remaining Objects
1	28	3	25
4	21	1	20
4	16	4	12
3	9	1	8
1	7	1	6
4	2	1	1
1	0		

After applying Exhibit V, it is found that the victory of NIM One-Three-Four is won by being able to leave by: $\mathbf{0\ mod\ 7\ or\ 2\ mod\ 7}$

2.6. NIM One-Three-Five Game Scheme

Suppose player A and player B have M objects placed on the table and the number of times to pick up objects is 1, 3 or 5 of the number of objects available. This game can be won by the player who is the last to bring/take the remaining objects.

Proof VI: (Player A starts first, but player B is the winner)

Two players take turns playing and removing 1, 3 or 5 objects from an array of 36 objects. The player who takes the last object is the winner. Here is the game flow that player A or player B can win the game.

Game Flow:

This game is won with a backward strategy. The second player (player B) can win by taking the remaining 1, 3, or 5 objects. The first player (player A) must set aside 1, 3, or 5 objects leaving 2 or 8 objects. Then player B in the last flow must remove 2 or 8 objects with 7,10,11,12,13, or 14 objects remaining while player A removes 10 or 15 objects from the array. Then player B must remove 10 or 15 objects on the second flow for player A with 13, 14, 19, 20, 23 or 24 objects left when player A has 16 or 25 objects left from the array. Then player B must remove 16 or 25 objects in the third flow for player A with 27, 30, 31, or 32 objects left when player A has 28 or 33 objects left from the pile of objects.

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Thus, the above illustration shows that player A can win the game by leaving 35, 32, 31, 30, 27, 24, 23, 20, 19, 14, and so on until 1 or 3 or 5 objects are left for player B. So player B can win the game by leaving as many objects:

$$\mathbf{0} \; \boldsymbol{mod} \; \mathbf{6} \; \boldsymbol{or} \; \mathbf{2} \; \boldsymbol{mod} \; \mathbf{6}$$

To understand the above evidence, consider table 6 below..

Table 6. Evidence of the game NIM One-Three-Five through 37 objects, the victory condition of the player who takes the object at the end is the winner (Player A starts first and is the winner).

Player A		Player B	
Objects Taken	Remaining Objects	Objects Taken	Remaining Objects
1	35	3	32
1	31	1	30
3	27	3	24
1	23	3	20
1	19	5	14
1	13	1	12
1	11	1	10
3	7	3	4
3	1	1	0

After applying Exhibit 6, it is found that the victory of NIM One-Three-Five is won by always leaving an object, namely $\mathbf{0} \; \boldsymbol{mod} \; \mathbf{6} \; \boldsymbol{or} \; \mathbf{2} \; \boldsymbol{mod} \; \mathbf{6}$

2.7. NIM One-Three-Five-Seven Game Scheme

There are 16 objects that are played by two players. The arrangement of these objects in the first row has one object, the second row has 3 objects, the third row has 5 objects, and the fourth row has 7 objects. The two players have alternating opportunities to play it. Players can freely take objects in the 1st, 2nd, 3rd or 4th row through the condition that they only take one and at most the remaining objects in each of these rows. And when taking objects, players may not take objects of different rows. The victory in this game is if the last player whose object takes the remaining objects, then the player loses and the player who leaves the object is the winner.

The losing player is if the sum of the base two (binary) numbers weighs 0 against the next player (Firdaus, 2011). It is known that base two is a set that has many but no pairs when the player has replaced a set of numbers with multiples. For example, multiples of 4, 2, and 1. The player performs initial calculations on all objects from each of the rows. Then, the player can estimate to change in multiples so that the player can form binary on those rows. The following is the form of

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counting objects in each row 1, 2, 3 and 4 are the number of objects with multiples of 4, 2 and 1 that will produce a binary value of 0.

Table 7. Binary sum results when converting into multiples of 4, 2, and 1.

Line 1=1	=1×1	=		1
Line 2=3	=1×2+1×1	=	2	1
Line 3=5	=1×4+1×1	= 4		1
Line 4=7	=1×4+1×2+1×1	= 4	2	1
The number of people who don't have a mate		0	0	0

Based on the table, there are four multiples of 1, two multiples of 2, two multiples of 4 then obtained by: 2 + 2 + 4 = 8 multiples, so the player has an even number of multiples, namely $8 \equiv 2\ (mod\ 2)$ because 8 | 2 or $8 \equiv 4\ (mod\ 2)$ because 8 | 4 has a remainder 0. In order for players to win this NIM game, they must make a move to make the enemy with the sum of base two numbers weighted 0, which is able to get pairs of multiples of 1, pairs of multiples of 2, and pairs of multiples of 4 and no one has a pair. By doing so, the opposing player has no winnings and each player must be able to use the enemy's carelessness to win.

Take a look at the following table to see how to make the sum of the two opponents' base numbers so that it has a weight of 0. For example, at the start of the game, the player picks 2 objects in the 3rd row, so the configuration can be written as follows.

Table 8. The shape of the NIM One-Three-Five-Seven game scheme configuration

Line 1=1	=1×1	=		1
Line 2=3	=1×2+1×1	=	2	1
Line 3=3	=1×2+1×1	=	2	1
Line 4=7	=1×4+1×2+1×1	= 4	2	1
The number of people who don't have a mate		1	1	0

By taking one multiple of 4 and one multiple of 2 by taking 6 objects in the fourth row, the player leaves the opponent with the configuration 1,3,3,1 where the next player is a weak position, because the addition of base two numbers by the next player is 0 where there are already multiples of 4 with their partners, multiples of 2 with their partners, multiples of 1 with their partners and none of them have no partner.

Table 9. Binary Summation Results of Configurations

Line 1=1	=1×1	=	1

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Volume 9, Nomor 2, Januari 2025
URL: http://ejournal.unma.ac.id/index.php/th

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Line 2=3	=1×2+1×1	=	2	1
Line 3=3	=1×2+1×1	=	2	1
Line 4=1	=1×1	=		1
The number of people who don't have a mate			0	0

Thus if a player has left for his match player produces a configuration of the sum of base two numbers weighted 0, so that the exact victory target for the players..

2.8. NIM Two-Four-Eight Game Scheme

There are 20 objects that are played by two players. The arrangement of the objects in the first row is 2 objects, the second row is 4 objects, the third row is 6 objects, and in the fourth row there are 8 objects. The two players have alternating opportunities to play it. The way to win is the same as the previous strategy. The losing player is if the sum of the base two numbers weighs 0 against the next player. For example, multiples of 4, 3, 2, and 1. The player first calculates all the objects and each row. Then, players can estimate to change in multiples so that players can form the base two numbers of each row. The following presents the objects in each row 1, 2, 3, and 4 are the number of multiples of 4, 3, 2, and 1 that will result in the number of base two numbers with a weight of 0.

Table 10. Binary sum results when converting into multiples of 4,3,2, and 1

Line 1=2	=1×2	=		2
Line 2=4	=1×1+1×3	=	3	1
Line 3=6	=1×1+1×3+1×2	= 1	3	2
Line 4=8	=1×4+1×4	= 4	4	
The number of people who don't have a mate		0	0	0

Based on the table, there are 2 multiples of 1, 2 multiples of 2, 3 multiples of 3 and 2 multiples of 4 then obtained by: 2 + 2 + 2 + 2 = 8, so the player has an even number of multiples, namely $8 \equiv 2 \ (mod\ 2)$ because 8 | 2 has a remainder 0. In order for players to win this NIM game, they must move to make the enemy with a binary sum result of 0, which is able to get a pair of multiples of 4, pairs of multiples of 3, pairs of multiples of 2, pairs of multiples of 1 and none of them are unpaired. By doing so, the opposing player has no winnings and each of the players must be able to use the carelessness of the enemy in order to win.

Take a look at the following table to see how to make the sum of the next two players' base numbers so that it has a weight of 0. For example, at the start of the game, a player picks 2 objects from row 1, so the configuration is written as follows.

Table 11. The shape of the NIM Two-Four-Nine-Eight game scheme configuration

Line 1=2	=1×2	=	2	
Line 2=4	=1×2+1×2	=	4	
Line 3=6	=1×3+1×3	=	3	3
line 4=8	=1×4+1×2+1×1	= 4	4	2
The number who don't have a partner		1	1	1

By taking 1 multiple of 4 through taking 4 objects in the 2nd row as a result the player leaves the next player with the configuration 2,2,3,4, Where in the next player is a weak position because the number of base two numbers by the next player weighs 0 which already has a multiple of 3 with its partner, a multiple of 2 with its partner and a pair of pairs of multiples of 3, pairs of multiples of 2 and none of them have no partner.

Table 12. Binary summation results of the Configuration

Line 1=2	=1×2	=		2
Line 2=2	=1×2+1×2	=	2	2
Line 3=3	=1×3+1×3	=	3	3
Line 4=4	=1×3+1×3+1×2	= 3	3	2
The number who don't have a partner		0	0	0

Thus if a player has left fos his match player produces a configuration of the sum of the base two numbers weighted 0, so that the exact victory target for the players.

CONCLUSIONS

From the explanation of the NIM game, it can be concluded that the winning strategy that can be used in the game to achieve victory is:

1. The Maximum NIM game obtained by the winning strategy is to leave the remaining objects that are congruent with 0 mod 5 or 0 mod (n + 1), where n = 4.
2. The NIM One-Four game obtained the winning strategy is to leave the remaining objects that are congruent with 0 mod 5 or 2 mod 5.
3. The Three-Five NIM game obtained the winning strategy is to leave the remaining objects or leave objects less than 3 that are congruent with 0 mod 8 or 2 mod 8.
4. The One-Three-Four NIM game is obtained by the winning strategy is to leave the remaining objects that are congruent with 0 mod 7 or 2 mod 7.

5. The One-Three-Five NIM game is obtained by the winning strategy is to leave the remaining objects that are congruent with 0 mod 6 or 2 mod 6.
6. The NIM One-Three-Five-Seven game obtained by the winning strategy is to leave the remaining paired objects on each object that are congruent with binary = 0
7. The Two-Four-Six-Eight NIM game obtained the winning strategy is to leave the remaining paired objects in each of its congruent objects with binary = 0

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