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From Learning Obstacles to Instructional Design: Bridging Theory and Practice in Algebra Education through Didactical Design Research

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ABSTRACT
This study aims to investigate the epistemological, ontogenic, and didactical obstacles encountered by junior secondary students in understanding algebraic operations and to propose a didactical design to overcome these barriers. Drawing on the Didactical Design Research (DDR) methodology, this qualitative research involved 25 Grade VII students and one mathematics teacher at MTs Negeri 1 Majalengka, West Java, during the 2024/2025 academic year. Data were collected through diagnostic tests, semi-structured interviews, and document analysis, employing purposive sampling to capture a range of student difficulties. The findings reveal that epistemological obstacles—rooted in students' prior arithmetic reasoning and symbolic misconceptions—were the most dominant, followed by ontogenic limitations in abstract thinking and didactical constraints embedded in instructional materials. Based on these insights, a Hypothetical Didactical Design (HDD) was constructed using contextual problems, semiotic tools (e.g., algebra tiles), reflective tasks, and explicit instructional focus on algebraic properties. The design facilitated students' shift from procedural to structural understanding of algebra, aligning instructional techniques with mathematical epistemology. The study contributes to bridging the gap between theoretical constructs and classroom practices in mathematics education, especially in addressing misconceptions through conceptually informed interventions. The paper includes 5 tables, 3 figures, and 58 references, offering theoretical grounding and empirical clarity for future studies, curriculum development, and teacher education.

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INTRODUCTION

Mathematics is considered a fundamental discipline in formal education, essential for cultivating students' logical, analytical, and problem-solving abilities. Among various topics, algebra plays a pivotal role as a bridge between arithmetic and more abstract mathematical concepts such as functions and calculus (Demonty, Vlassis, & Fagnant, 2018). In secondary education, particularly at the junior high school level, mastery of algebraic operations is crucial not only for academic progression but also for the development of abstract thinking skills (Jin & Wong, 2015). However, despite its centrality, many students experience persistent difficulties in learning algebra, especially in understanding and applying algebraic operations. These difficulties often arise not from the absence of procedural skills but from deeper conceptual barriers that hinder meaningful understanding, commonly referred to as learning obstacles (Jupri, Drijvers, & van den Heuvel-Panhuizen, 2014; Russell, O'Dwyer, & Miranda, 2009). Misconceptions, limited representational fluency, and inadequate instructional approaches further exacerbate these challenges (Witzel, Mercer, & Miller, 2003; Jin & Wong, 2015), underscoring the need for pedagogical interventions that prioritise conceptual clarity and cognitive accessibility.

Recent studies further emphasize that these algebraic difficulties often manifest in the form of symbolic misinterpretations, procedural errors, and overgeneralized rules derived from arithmetic (Booth & Koedinger, 2008; Pramesti & Retnawati, 2019; Veith, Bitzenbauer, & Girnat, 2022). While some of these errors are linked to instructional shortcomings, deeper analysis reveals that they are frequently rooted in students' epistemological frameworks—namely, their implicit beliefs and cognitive structures concerning how mathematical knowledge is acquired and justified (Brown, 2008; Schneider, 2014). These obstacles inhibit the transition from concrete arithmetic reasoning to abstract algebraic thinking, making conceptual understanding difficult to attain (Moru, 2007; Schneider, 2014).

In this regard, epistemological obstacles—conceptual barriers originating from prior knowledge that resist transformation—have become a growing focus in mathematics education research. Drawing on Brousseau's Theory of Didactical Situations and further developed by Balacheff (2008, 2009), epistemological obstacles are distinguished from ontogenic and didactical ones through their deep cognitive roots and resistance to new knowledge (Brousseau, 2002; Herscovics, 2018). Despite this theoretical development, empirical investigations focusing specifically on epistemological obstacles within junior secondary algebra remain limited (Subroto & Suryadi, 2018; Hersant & Perrin-Glorian, 2005).

Several recent studies support the urgency of exploring this issue further. Nansiana, Usodo, and Fitriana (2024) revealed that students' persistent reliance on arithmetic reasoning disrupted their engagement with variables and symbolic manipulation, indicating unresolved epistemological conflicts. Similarly, Utami and Prabawanto (2023), through a systematic literature review, identified a gap in empirical studies that link these obstacles directly to patterns of student error in algebra. Studies by Sidik, Suryadi, and Turmudi (2021) as well as Fauziah, Lidinillah, and Apriani (2023) also confirm that students frequently misinterpret algebraic symbols due to informal strategies rooted in primary-level learning. Nevertheless, despite the theoretical depth of the epistemological framework, its practical application in classroom instruction remains minimal (Balacheff, 2009), indicating a significant gap in translating theory into didactical practice.

Therefore, there is a pressing need for research that not only identifies and characterizes epistemological obstacles in algebraic learning but also formulates didactical approaches that address these barriers within authentic classroom contexts. Such efforts would bridge the divide between theory and practice in mathematics education, particularly in enhancing students' conceptual understanding and algebraic thinking at the junior secondary level. This study seeks to fill that gap by investigating how epistemological obstacles hinder students' conceptual understanding of algebraic operations in junior secondary mathematics. By identifying these obstacles through written assessments, interviews, and document analysis, this research aims to inform the development of a didactical design tailored to overcome such barriers. Specifically, the research addresses two guiding questions: (1) *What epistemological obstacles do students face in understanding algebraic operations?* Moreover, (2) *What didactical design can be proposed to minimise these obstacles?* The novelty of this study lies in its integration of Didactical Design Research (DDR) methodology with the theoretical framework of epistemological obstacles, generating pedagogical interventions that are theoretically grounded and contextually relevant for algebra instruction at the junior secondary level.

METHODS

This research employed a qualitative approach with the Didactical Design Research (DDR) methodology developed by Suryadi (2019). DDR is designed to analyze and construct didactical situations by identifying learning obstacles encountered by students and providing alternative instructional designs. This research focused on the first phase of DDR, namely the prospective analysis, which includes analyzing the learning content (scholarly knowledge), identifying learning obstacles, and proposing a hypothetical didactical design (HDD). The study was conducted at MTs Negeri 1 Majalengka, West Java, during the second semester of the 2024/2025 academic year. The participants were 25 students from class VII who had completed instruction on algebraic operations, along with one mathematics teacher. The participants were selected using purposive sampling, focusing on students with varied academic achievement to capture a diverse range of obstacles.

The research followed the prospective analysis phase of DDR, which consists of the following stages:

1. Identification of Learning Obstacles
   Student difficulties were identified through written diagnostic tests, in-depth student interviews, and analysis of learning documents (e.g., textbooks, lesson plans). The data helped classify obstacles into epistemological, ontogenic, and didactical types (Brousseau & Balacheff, 1997).
2. Analysis of Scholarly Knowledge and Knowledge to be Taught
   The concept of algebraic operations was analysed at two levels:
   - Scholarly knowledge: explored through textbooks used in undergraduate mathematics (Herstein, 2006), particularly group theory and algebraic structures.
   - Knowledge to be taught: analysed from the 2022 Indonesian mathematics curriculum textbook for Grade VII.
3. Design of Hypothetical Didactical Design (HDD)

Based on the identified obstacles, a hypothetical didactical design was formulated as an alternative instructional sequence aimed at minimising the learning difficulties observed.

To ensure data triangulation and credibility, the following data collection methods were applied:

1. Diagnostic Test
   A written test consisting of open-ended problems was administered to assess students' understanding of algebraic operations. The items were designed to reveal misconceptions and reasoning processes.
2. Semi-Structured Interviews
   Interviews were conducted with six students selected based on their test responses to clarify reasoning errors and uncover conceptual misunderstandings. A mathematics teacher was also interviewed to obtain additional perspectives on instructional methods and student learning experiences.
3. Document Analysis
   Learning materials such as lesson plans, textbooks, and student worksheets were analyzed to identify potential didactical obstacles embedded in instructional design.

The collected data were analyzed using the interactive model of Miles and Huberman (Sugiyono, 2013), which consists of three main components:

1. Data Reduction: selecting and summarizing relevant information from tests, interviews, and documents to focus on evidence of epistemological obstacles.
2. Data Display: organizing data into tables, figures, and narrative descriptions to support interpretation.
3. Conclusion Drawing and Verification: synthesizing findings to answer research questions and validate results through cross-referencing between data sources (triangulation).

The research process was iterative and reflexive, allowing the researcher to revisit and refine the findings based on emerging themes from the data.

RESULTS AND DISCUSSION

Result

Identified Learning Obstacles in Algebraic Operations

The results of diagnostic tests and student interviews revealed that many students encountered significant difficulties in performing algebraic operations. These difficulties were categorized into three types of learning obstacles as proposed by Brousseau and Balacheff (1997): epistemological, ontogenic, and didactical obstacles. These error patterns, observed across a substantial portion of the sample, indicate a systemic misunderstanding of fundamental algebraic operations. To further illustrate the distribution of these misconceptions, the following figure summarizes the types of errors committed by students during the diagnostic test.

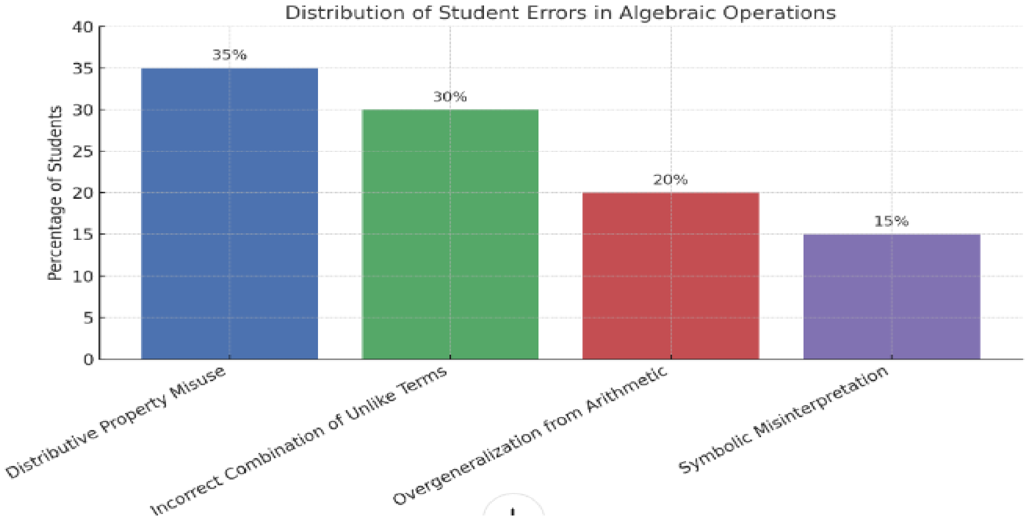

Figure 1. Distribution of Student Errors in Algebraic Operations.

As shown in Figure 1, the most prevalent error involved misapplying the distributive property, followed closely by incorrectly combining unlike terms. These findings suggest that students' difficulties stem not merely from a lack of procedural knowledge but from conceptual misconceptions deeply rooted in prior learning. To explore these misconceptions more deeply, the following section focuses on the epistemological obstacles that emerged as the dominant learning barrier in students' algebraic reasoning.

1. *Epistemological Obstacles*

The diagnostic test administered to thirty students revealed consistent misconceptions in algebraic operations, particularly in the use of the distributive property and the identification of like terms. One prominent error pattern was observed when simplifying expressions such as $2(x + 3)$, Which many students reduced to $2x + 3$, Omitting the multiplication of the constant term. Similarly, in expressions like $4a + 2b - 3a + b$, Students combined terms indiscriminately to yield incorrect results, such as 3ab or $4a - 3a + 2b + b = 3ab$, Indicating a failure to recognise algebraic structure and symbolic conventions. These errors suggest that students were operating under arithmetic-based reasoning frameworks, treating variables as numerical values and ignoring algebraic principles of term classification and distribution.

Another illustrative example of epistemological obstacles was evident in students' attempts to apply the distributive property in purely arithmetic contexts. In one case (Figure 2), a student decomposed the numerical expression. $5 + 5 + 5 + 5 + 4$ Into a form involving distribution: $-2(5 + 2) + 2 \cdot 5$. While the final sum arrived at $45 + 4$ It was numerically correct, but the process relied on arbitrary and unjustified manipulation of number groupings. This suggests a misconception of the distributive law, where the student treats it as a general-purpose tool for rearranging numbers rather than a structurally defined operation within algebra. Such reasoning reveals a persistent epistemological obstacle, wherein symbolic operations are imitated procedurally without understanding their mathematical constraints or applicability.

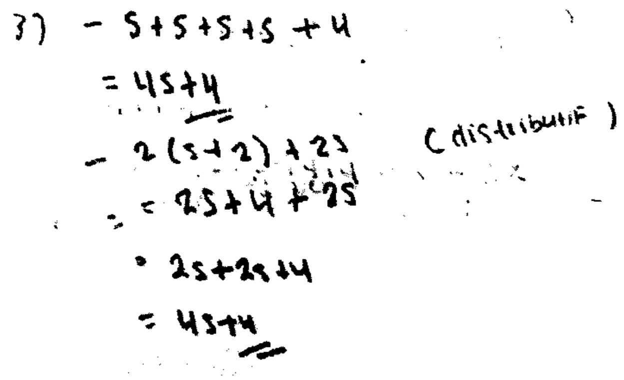

Figure 2. Example of a student misapplying the distributive property in an arithmetic context, revealing an epistemological obstacle of algebraic structure.

The interview with student DNAN revealed a divergence between perceived ease and actual conceptual understanding. Although the student claimed the problem was easy and reported no difficulties, they failed to identify or articulate any algebraic properties involved in the solution. When asked about this omission, the student cited time pressure and confusion as reasons for not stating the underlying rules. Notably, the student asserted a general understanding of algebraic properties ("Insya Allah paham"), despite being unable to apply them in context. This response illustrates a characteristic epistemological obstacle, in which students possess a superficial or procedural grasp of algebra but lack explicit and structured conceptual frameworks. The inability to recognize or name the properties in use suggests that their algebraic reasoning remains informal and undeveloped.

Table 1. Thematic Analysis and Its Relation to Epistemological Obstacles

Interview Finding	Educational Meaning	Relation to Epistemological Obstacle
The problem was perceived as easy, but the student could not explain the conceptual steps.	The student imitates procedures without a clear understanding of the underlying concepts.	Operational understanding without structural comprehension.
The student mentioned being rushed and did not state the algebraic properties involved.	Indicates weak internalization of algebraic principles.	Inability to access or activate conceptual knowledge when needed.
Student claimed to "understand," but failed to justify the reasoning.	There is a gap between confidence and conceptual competence.	Understanding is unstructured and lacks reflective depth.

Insights from Student Interviews

Semi-structured interviews with three selected students—DNAN, AN, and KMA—provided in-depth evidence of epistemological obstacles in learning algebra. Student DNAN was able to carry out algebraic procedures but struggled to explain the properties underlying each step. The student admitted being "in a rush," yet further probing revealed a superficial understanding of symbolic manipulation, suggesting reliance on memorized procedures without structural insight. Student AN faced significant difficulty in translating a word problem into an algebraic expression. Although the student understood the context, they were unable to represent it symbolically and admitted to guessing the solution, highlighting a representational obstacle. Meanwhile, student KMA exhibited confusion when distinguishing variables, coefficients, and constants. Despite partially completing the task, the student hesitated when asked to name algebraic elements and expressed difficulty with fractional operations, indicating both terminological uncertainty and arithmetic interference. Across all three cases, students demonstrated fragmented conceptual understanding and procedural actions unaccompanied by reflective reasoning. Their inability to articulate the logic behind their work points to deeper cognitive constraints shaped by prior arithmetic experiences, consistent with the features of epistemological obstacles in early algebra learning.

Table 2. Justification Table: Thematic Summary of Student Interview Insights

Student	Observed Behavior	Interview Quote (Evidence)	Interpretation
DNAN	Performed procedures correctly, but could not explain the properties used	*"Saya bingung Bu... saya jadi terburu-buru mengerjakannya.""InsyaAllah paham Bu, cuman karena terburu-buru saya jadi bingung menentukan sifat-sifatnya."*	Procedural fluency without structural understanding; evidence of superficial reasoning and limited metacognition.
AN	Understood the story context but failed to express it in algebraic form	*"Ngerti, tapi bingung buat ubah ke bentuk aljabar.""Soalnya hanya asal mengerjakan, jadi takut salah."*	Representational epistemological obstacle; difficulty translating semantic to symbolic representation.
KMA	Could manipulate symbols partially, but was confused about variables and coefficients.	*"Koefisiennya disamakan... eh dikumpulkan.""Gatau Bu, saya lupa lagi nama nya.""Saya masih suka bingung."*	Terminological fragility and arithmetic interference; conceptual boundaries between terms, variables, and coefficients remain unclear.

Observations from Document Analysis

To uncover potential instructional roots of these obstacles, lesson plans, mathematics textbooks (Grade VII), and student worksheets were examined. The document analysis revealed that most instructional materials emphasized procedural fluency, such as "follow the rule to combine like terms," without sufficient conceptual scaffolding. For example, textbooks provided multiple procedural examples without explicitly discussing the rationale for combining terms or visual models to illustrate distributive operations. Additionally, symbolic representations dominated the materials, with minimal use of contextual or visual tools (e.g., algebra tiles or diagrams). The teacher's lesson plan focused on drill-based learning and lacked formative assessment checkpoints that could address misconceptions. These features are indicative of didactical obstacles, where the way knowledge is structured and presented impedes students' conceptual understanding.

2. *Ontogenic Obstacles*

Ontogenic obstacles refer to learning difficulties that stem from students' developmental readiness, particularly in transitioning from concrete arithmetic reasoning to abstract symbolic thinking (Suryadi, 2019). In the context of algebraic operations, several students in this study demonstrated such cognitive limitations. For example, student AN expressed difficulty with algebraic modeling despite understanding the problem context, stating, "*I understand the question, but I am confused how to turn it into algebra.*" Similarly, another student remarked, "*Letters in math are confusing,*" reflecting a lack of symbolic familiarity.

These responses illustrate a typical developmental phase in early algebra learning, in which students view mathematical symbols as disconnected or foreign rather than as generalized numerical entities. Limited arithmetic fluency further compounds this symbolic uncertainty. Students who had not yet mastered basic operations, such as multiplication, factoring, or handling negative numbers, struggled with tasks requiring those operations as foundational. One common error observed was interpreting -3×2 as -5 or 6, which hindered their application of the distributive property and accurate symbolic manipulation. Such calculation errors suggest undeveloped number sense and a lack of flexibility in coordinating numerical and symbolic representations. While some students could perform isolated procedures, they lacked the cognitive maturity to synthesize these operations within coherent algebraic reasoning. This reflects the essence of ontogenic obstacles: a misalignment between procedural exposure and conceptual development, rooted in the learner's stage of cognitive abstraction. Addressing these difficulties necessitates instructional scaffolds that bridge concrete and abstract reasoning. Strategies such as using algebra tiles, contextual problem scenarios, and visual representations can support this transition. Without such support, students risk interpreting algebra as a disjointed sequence of symbol-based tasks rather than as a meaningful extension of arithmetic.

3. *Didactical Obstacles*

The analysis of instructional documents and teacher interviews revealed the presence of didactical obstacles—barriers arising from the way knowledge is structured and delivered in the classroom. These obstacles were evident in both the instructional materials and the students' written work. One illustrative example (see Figure 3) demonstrates how students may develop procedural fluency without conceptual grounding.

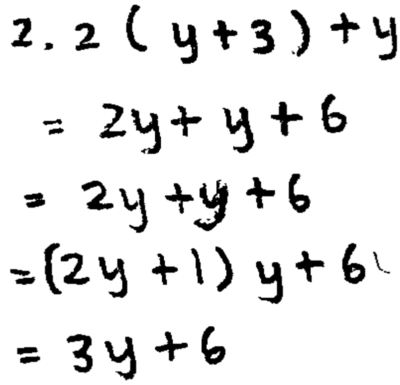

Figure 3. Student's Misapplication of Algebraic Structure Due to Didactical Obstacle

In this example, the student initially applies the distributive property correctly:

$$2.2(y + 3) + y = 2y + y + 6$$

However, in a subsequent step, the student writes:

$$(2y + 1)y + 6$$

This manipulation is structurally invalid in algebra. By reintroducing y into the expression $(2y + 1)y$, The student demonstrates a lack of understanding that $2y + y$ is already a simplified linear term, and not a multiplicative expression. This kind of reasoning indicates symbolic misinterpretation, likely resulting from instruction that overemphasizes procedural steps (e.g., "combine like terms" or "multiply before adding") without making explicit the underlying algebraic structures.

This finding aligns with classroom observations and textbook analysis, which revealed a heavy reliance on symbolic representation and drill-based examples, often devoid of conceptual explanation or visual modeling (e.g., algebra tiles or contextual illustrations). The student's error in Figure 3 underscores a common pattern: symbolic manipulation is applied algorithmically, without sufficient attention to the meaning or validity of the transformation. In line with Brousseau and Balacheff's (1997) framework, such errors represent didactical obstacles, not simply misunderstandings on the part of students, but conceptual gaps produced or reinforced by the didactic system itself. When algebra is taught primarily as a set of symbolic procedures, students may internalize rules without understanding their domain of validity, leading to rigid application even when structurally inappropriate. Thus, instructional approaches that lack formative feedback, conceptual scaffolding, and opportunities for reflection contribute significantly to the persistence of these misconceptions. Addressing such obstacles requires pedagogical shifts toward meaning-oriented learning, the use of multiple representations, and the incorporation of error-based discussions to develop students' metacognitive awareness of algebraic reasoning.

4. Analysis of Scholarly Knowledge and Knowledge to be Taught

An analysis of scholarly knowledge and the knowledge taught in school was conducted to support the identification of learning obstacles. In tertiary-level mathematics, algebraic operations are situated within the formal framework of abstract algebra, specifically group theory, where operations are governed by axioms such as closure, associativity, identity, and inverses (Herstein, 2006). These axioms provide a rigorous foundation for understanding the behavior of algebraic structures and symbolic manipulation.

In contrast, the junior secondary school curriculum simplifies these structures into rule-based procedures—such as combining like terms, simplifying expressions, and using the distributive property—without explicitly introducing their underlying logical or structural foundations. This transformation of mathematical knowledge through the process of didactical transposition creates a cognitive gap between the mathematical essence of algebra and its classroom presentation. Students are thus introduced to symbols and procedures without sufficient grounding in their structural meaning, encouraging rote application rather than conceptual understanding. Consequently, algebraic expressions are often interpreted as disconnected objects rather than elements within a relational and logical system. This gap may foster epistemological obstacles by obscuring the purpose and structure behind symbolic manipulation.

To illustrate this shift, Table 1 presents a simplified comparison between scholarly knowledge and school-level representations of algebraic operations:

Table 3. Comparison Between Scholarly Knowledge and Knowledge to Be Taught in Algebra

Dimension	Scholarly Knowledge (University Level)	Knowledge Taught (Junior Secondary)	Potential Learning Obstacle
Conceptual Foundation	Based on group theory, the axioms of operations	Procedural rules (e.g., simplify terms)	Lack of awareness of why operations work
Symbolic Meaning	Symbols represent elements in an algebraic structure	Symbols are often treated as labels or tokens	Misinterpretation of symbols as disconnected items
Purpose of Operation	Formal proof and generalization	Procedural accuracy (e.g., correct answer)	No clear understanding of "why" simplification is done
Example	Demonstrate associativity: $(a \bullet b) \bullet c = a \bullet (b \bullet c)$	Apply: $(2x + 3x = 5x)$	Students cannot explain why terms can be combined
Representation of Variables	Variables as general elements of sets/groups	Variables are often treated like unknowns in arithmetic	Overgeneralization from arithmetic to algebra

5. Proposed Hypothetical Didactical Design (HDD)

Based on triangulated data obtained from diagnostic test results, student interviews, and document analysis, a Hypothetical Didactical Design (HDD) was constructed to address the observed epistemological, ontogenic, and didactical obstacles encountered by students in learning algebraic operations. This design is conceptually grounded in the Anthropological Theory of the Didactic (ATD) and operationalized through the principles of Didactical Design Research (DDR), allowing the instructional sequence to be context-sensitive and theoretically robust.

The primary goal of the HDD is to facilitate students' cognitive progression from arithmetic-based reasoning—where operations are understood concretely and procedurally—to formal algebraic thinking that demands structural awareness and symbolic generalization. The design aims to alleviate misconceptions commonly associated with symbolic manipulation, variable interpretation, and algebraic properties such as the distributive and associative laws. Furthermore, the HDD seeks to bridge the divide between procedural fluency and conceptual understanding through didactical scaffolding, visual representations, and reflective tasks that encourage metacognitive engagement.

By integrating semiotic resources (such as algebra tiles and diagrams) and embedding contrastive tasks that challenge students' prior assumptions, the proposed design functions not only as a remediation framework but also as a preventive pedagogical strategy. It promotes structural insight at the initial stages of algebra learning, thus addressing epistemological obstacles before they become deeply entrenched. Ultimately, this didactical intervention aspires to cultivate a more coherent and meaningful understanding of algebra among junior secondary students.

Table 4. Didactical Elements and Their Roles in Addressing Learning Obstacles

Component	Description	Targeted Obstacle	Implementation Notes
Contextualized Problem Situations	Problems rooted in real-life contexts (e.g., shopping scenarios, area models) to introduce variables meaningfully.	Ontogenic obstacle: lack of abstraction readiness	Begin with concrete quantities that naturally map onto algebraic expressions.
Visual Tools and Manipulatives	Use of algebra tiles, semantic mapping, and diagrams to represent distribution and like terms visually.	Epistemological obstacle: misinterpretation of structure	Tools are used in parallel with symbolic expressions to support dual coding.
Explicit Focus on Algebraic Properties	Structured activities that introduce and label distributive, associative, and commutative properties.	Didactical obstacle: lack of emphasis on conceptual rules in textbooks	Properties are highlighted and discussed in every step of simplification.
Contrastive Analysis Tasks	Students analyze and reflect on incorrect simplifications and explain *why* they are wrong.	Epistemological obstacle: overgeneralization from arithmetic	Encourages metacognitive awareness of symbolic conventions.
Scaffolded Practice Sequences	Tasks gradually shift from guided to independent, incorporating varying symbolic complexity.	Ontogenic and epistemological	Include prompts for students to identify operations and properties used.
Reflexive Journals or Meta-Talk	Students verbalize their reasoning and strategies after each task.	All types of obstacles	Encourages internalization and conceptual articulation.

Following the identification of key instructional components and their alignment with targeted learning obstacles (as outlined in Table 4), Table 5 presents the practical progression of the Hypothetical Didactical Design (HDD) across a six-session instructional sequence. This weekly-level structure integrates didactical principles from the Anthropological Theory of the Didactic (ATD) and Didactical Design Research (DDR), illustrating how each instructional activity is enacted within a coherent pedagogical flow.

Table 5. Practical Flow (Weekly Lesson-Level)

Session	Activity Focus	Instructional Strategy
1	Introduction to variables via contextual stories	Didactical situation through narrative modeling (transposition contextualisée vers le savoir enseigné)
2	Combining like terms using color-coded algebra tiles	Semiotic mediation via artefacts and institutional co-construction of meaning
3	Distributive property with pictorial support	Dual semiotic representation with institutionalized labeling
4	Common student errors: analysis and correction	Didactical milieu for dialectical devolution through collective error reflection
5	Symbolic tasks: transition to pure algebra	Progressive institutionalization through graduated didactical support
6	Reflection and journaling	Metacognitive regulation within praxeological reflection tasks

Discussion

This study aimed to identify, analyze, and remediate learning obstacles in algebraic operations among junior secondary students through the lens of Didactical Design Research (DDR). Drawing upon the theoretical frameworks of the Anthropological Theory of the Didactic (ATD), the Theory of Didactical Situations (TDS), and Didactical Transposition, the findings contribute to a more nuanced understanding of the sources and persistence of algebraic misconceptions. In what follows, the results are interpreted in light of these frameworks and situated within the context of relevant prior literature.

A central finding of this study was that students' errors were not merely procedural, but deeply rooted in epistemological obstacles. Consistent with Dewi, Mahani, and Wijayanti (2021) and Maarif, Setiarini, and Nurafni (2020), students demonstrated misconceptions stemming from a praxeological gap—where techniques taught in class were disconnected from the theoretical rationales that justify them (Ramdhani, Suryadi, & Prabawanto, 2021). This observation supports the position of Artigue (2009), who argues that such obstacles can serve as productive entry points within DDR for constructing Hypothetical Didactical Designs (HDDs) aimed at resolving these ruptures.

The HDD developed in this study functioned both as a pedagogical tool and a research instrument, structured to align with DDR principles. This aligns with previous research advocating the use of didactical design to reconstruct fragmented student understanding through epistemologically informed tasks (Annizar & Suryadi, 2016; Jatmiko, Herman, & Dahlan, 2021; Pratamawati, 2020). The study further identified ontogenic and didactical obstacles, including limitations in students' developmental readiness and misaligned instructional strategies (Ulfa, Jupri, & Turmudi, 2021), reinforcing the need for didactical engineering that is sensitive to both cognitive development and institutional context.

For instance, student confusion over symbolic rules—such as the distributive property and combining unlike terms—revealed how procedural instruction, when decoupled from conceptual foundations, can reinforce epistemic ruptures. As DDR posits, these discontinuities between technique and theory warrant intentional design responses. In this study, tasks integrating semiotic tools, contrastive representations, and metacognitive prompts were used to foster reflective awareness and support the transition toward conceptual understanding.

Another key theme that emerged relates to the impact of didactical transposition. The comparative analysis between formal abstract algebra and school-level procedural practice (see Table 3) revealed a clear epistemological gap, whereby school mathematics fails to reflect the theoretical rigor of the discipline (Chevallard, 1989; Chevallard & Bosch, 2014). As Kang and Kilpatrick (1992) and Pansell (2023) observed, higher mathematics frames operations within formal systems, yet classroom practice often simplifies them into procedural heuristics devoid of meaning.

This discrepancy supports Chevallard's (1989) notion of *le savoir en jeu*, where the "knowledge at stake" is obscured through instructional reduction. In alignment with findings from Mensah (2025) and Gueudet, Doukhan, and Quéré (2022), the current study demonstrates that such reductions prompt students to mimic techniques without understanding the rationale behind them—further reinforcing epistemological obstacles and undermining the coherence of algebraic knowledge.

TDS offers additional insight into these findings. According to Brousseau (1997), meaningful mathematical learning occurs in a milieu that allows students to engage with and adapt to structured learning environments. However, the study found that students' prior learning environments lacked such conditions, as tasks often emphasized algorithmic repetition over problem exploration (Inayah, 2018; Fuadiah, 2017). As a result, students missed critical opportunities for hypothesis testing and conceptual construction.

In response, the HDD deliberately incorporated adidactical situations in which students could experiment, make errors, and revise their thinking independently—an application of TDS principles as advocated by Putra et al. (2017) and Nurwani and Putra (2017). Activities such as visual modeling and error analysis activated key TDS phases, particularly *devolution* and *institutionalization*, enabling the teacher to step back and later consolidate emerging knowledge (Ria, Lusiana, & Fuadiah, 2023). These findings are consistent with previous work highlighting the importance of engineered didactical situations for provoking conceptual change (Suryadi, 2013; Purnomo et al., 2024).

Praxeological coherence was also advanced by embedding tasks that linked operational techniques to theoretical justifications, facilitating students' transition from arithmetic to symbolic reasoning. This approach not only fostered structural algebraic understanding but also exemplified the DDR goal of synchronizing pedagogical design with theoretical constructs.

Ontogenic obstacles were another significant theme. Defined by Brousseau and Balacheff (1997) as challenges rooted in the learner's developmental stage, these were evident in students' difficulties distinguishing between variables and constants, their misinterpretation of symbolic structures, and their inconsistent application of algebraic operations. These challenges mirror findings from Natalia, Kusumah, and Ditasona (2023), Lutfi, Juandi, and Jupri (2021), and Shahrul, Nasir, and Suryadi (2025), all of whom noted the cognitive demands associated with early algebra learning. This difficulty has also been previously reported in broader contexts (Kamii, 1990; Carraher et al., 2006).

To address these ontogenic challenges, the HDD incorporated semiotic and contextual scaffolds—such as algebra tiles, diagrammatic models, and narrative tasks—particularly during the early stages of instruction. These tools were more than visual aids; they supported instrumental genesis, a process in which students internalize tools into cognitive schemas, as theorized by Rabardel. This aligns with Vygotskian views of mediated learning and corresponds with DDR's concern for representational and cognitive affordances (Pertiwi, Dasari, & Sumiaty, 2023).

In addition, the study applied principles of instrumental orchestration (Trouche, 2004), wherein teacher interventions, artefacts, and task sequences were deliberately coordinated. This strategy enabled differentiation according to students' symbolic fluency, arithmetic proficiency, and structural understanding—accommodating asynchronous development and supporting the gradual internalization of formal algebraic knowledge.

In summary, the study demonstrates that effective algebra instruction must move beyond procedural transmission and instead be grounded in didactical designs that respond to the complex interplay of cognitive, ontogenic, and institutional dynamics. The DDR framework offers a powerful means for constructing such interventions, while the integration of ATD and TDS supports the creation of theoretically principled and contextually responsive didactical situations. Pedagogically, the study advocates for recontextualizing algebra within meaningful, real-world problems, making procedural rules explicitly linked to underlying concepts, and encouraging reflective error analysis. Theoretically, the findings contribute to ongoing efforts to operationalize DDR within the ATD–TDS paradigm, demonstrating how empirical insights can inform coherent, theory-driven didactical innovations.

CONCLUSION

This study investigated students' learning obstacles in algebraic operations using the Didactical Design Research (DDR) approach, focusing on epistemological, ontogenic, and didactical dimensions. The findings revealed that students' misconceptions were rooted not only in procedural gaps but also in deeper cognitive structures shaped by prior arithmetic reasoning. Epistemological obstacles—such as misuse of the distributive property and symbolic confusion—emerged as dominant, reflecting a rupture in the didactical transposition from scholarly algebra to classroom practice. Ontogenic limitations and didactical shortcomings further compounded these difficulties, restricting students' conceptual development. In response, a Hypothetical Didactical Design (HDD) was proposed, grounded in the Anthropological Theory of the Didactic (ATD) and the Theory of Didactical Situations (TDS). The HDD integrates contextual modeling, semiotic tools, contrastive tasks, and reflective strategies to support the transition from arithmetic-based procedures to structural algebraic understanding. The study advances DDR by illustrating how empirical insight into learning obstacles can inform theoretically robust and context-sensitive pedagogical designs. Nevertheless, this research was limited to the prospective phase and a single instructional context. Future studies should explore the implementation and refinement of the HDD across diverse settings and topics to validate its broader applicability. Sustained efforts to bridge epistemological gaps in algebra must prioritize designs that are both conceptually coherent and developmentally responsive.

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