Volume 8, Number 2, October 2026
Jurnal Didactical Mathematics
DOI: https://doi.org/10.31949/dm.v8i2.18499
p-ISSN: 2622–7525, e-ISSN: 2654-9417

From Multiplicative Growth to Institutionalized Exponential Knowledge: A Praxeological Analysis of Exponential Learning in a Secondary Mathematics Textbook

Laila Cantika[1], Septiani Yugni Maudy[2], Mulia Putra[3]

[1,2,3] Mathematics Education Study Program, Faculty of Teacher Training and Education, Universitas Singaperbangsa Karawang, Karawang, Indonesia

Corresponding Author: 2210631050018@Student.unsika.ac.id

Article Info

Article History
Submitted: 03-06-2026
Revised: 11-06-2026
Accepted: 13-06-2026
Published: 13-06-2026

Keywords:

Exponentiation; Praxeology; Mathematics textbook; Anthropological theory of the didactic; Mathematical knowledge

Abstract

This study investigates the praxeological organization of exponent learning in a Grade 10 mathematics textbook through the lens of the Anthropological Theory of the Didactic (ATD). The study aims to examine how exponent knowledge is introduced, developed, and institutionalized through the relationships among tasks, techniques, technologies, and theories. A qualitative document analysis was conducted using the exponent chapter of the 2021 Indonesian Mathematics Electronic School Book. The analysis focused on Exploration 1.1, Exploration 1.2, and Exercise 1.1, which cover the initial development of concepts of exponents. Mathematical tasks were identified and reconstructed into local praxeologies, which were subsequently synthesized into a global praxeological organization. The findings reveal a coherent learning trajectory consisting of three successive phases. Exploration 1.1 constructs exponentiation through multiplicative growth situations, enabling students to develop meaning from repeated multiplication. Exploration 1.2 extends this understanding by identifying and generalizing the properties of exponents. Exercise 1.1 institutionalizes exponent knowledge by transforming exponent properties into tools for justification, equation solving, and symbolic simplification. The reconstructed global organization demonstrates a progression from meaning construction to formal mathematical practice. The analysis further shows that the visibility of the praxis block increases throughout the chapter, whereas technological explanations become less explicit during the institutionalization phase. These findings contribute to textbook research by illustrating how the sequencing of praxeological components shapes students' opportunities to construct, generalize, and apply mathematical knowledge. The study also highlights the value of ATD as a framework for examining the epistemological organization of mathematical content in school textbooks.

INTRODUCTION

Developing students' conceptual understanding has become a central objective of contemporary mathematics education because it enables learners to establish meaningful connections among mathematical ideas, explain relationships between concepts, and apply knowledge flexibly across different contexts. A substantial body of research has demonstrated that conceptual understanding plays a critical role in supporting mathematical reasoning, representation, problem-solving, and long-term knowledge retention (Vinner, 2020; Kowiyah et al., 2019; Borji et al., 2021; Bani Irshid et al., 2023). In contrast, learning experiences that emphasize procedural knowledge alone often allow students to execute mathematical operations correctly without fully understanding the underlying principles that justify those procedures. Such procedural performance is frequently context-dependent and difficult to transfer to unfamiliar situations. Recent studies further suggest that conceptual understanding and procedural fluency should not be regarded as competing forms of knowledge but rather as complementary dimensions of mathematical proficiency. Durable procedural fluency is more likely to emerge when students understand the mathematical meanings underlying procedures rather than merely memorizing rules and algorithms (Keazer & Phaiah, 2023; Polly & Martin, 2025). Consequently, effective mathematics instruction should provide opportunities for students not only to perform procedures but also to explain, justify, and connect those procedures to broader mathematical ideas, making conceptual understanding an essential foundation for meaningful and sustainable mathematical learning.

One mathematical topic that particularly illustrates the importance of conceptual understanding is exponentiation. Exponents are a foundational concept in secondary mathematics because they underpin subsequent learning in algebraic reasoning, exponential functions, logarithms, scientific notation, and advanced mathematical modeling. Understanding exponentiation requires students not only to manipulate symbolic expressions but also to recognize exponents as representations of multiplicative relationships and repeated growth processes. However, previous studies have consistently reported that students experience substantial difficulties in interpreting exponent notation, understanding exponent laws, and coordinating symbolic procedures with underlying mathematical meanings (Ulusoy, 2019; Syafiqoh et al., 2018; Şenay, 2024). Similar findings have been reported in the Indonesian context, where learning obstacles continue to emerge even among students in higher education (Suarka & Kusumah, 2024). Furthermore, misconceptions associated with exponentiation extend beyond symbolic manipulation and frequently affect students' interpretations of exponential growth represented graphically (Ciccione et al., 2022). These findings suggest that difficulties in learning exponents are closely associated with limited opportunities to develop conceptual understanding and to access the mathematical justifications underlying exponent procedures.

From a learning perspective, exponentiation may be viewed as a progressive development of mathematical meaning. Students initially encounter exponential relationships in contexts involving repeated multiplication and multiplicative growth. Through subsequent learning activities, they identify regularities within exponent operations, formulate generalized rules, and eventually employ these rules as formal mathematical tools for reasoning and problem-solving. This progression reflects a movement from contextual interpretation toward increasingly abstract and institutionalized mathematical knowledge. Consequently, examining how instructional materials organize this progression becomes important for understanding the learning opportunities available to students and the extent to which those opportunities support conceptual understanding alongside procedural fluency.

The opportunities available for developing conceptual understanding are strongly influenced by the instructional resources used in classrooms. Among these resources, textbooks play a particularly important role because they often determine how mathematical concepts are introduced, sequenced, explained, and practiced. Textbooks not only present mathematical content but also communicate particular views of mathematical knowledge, including what counts as legitimate mathematical activity and how mathematical ideas should be justified. Consequently, textbook analysis has become an important area of research in mathematics education, as it provides insights into the learning opportunities available to students (Bittar, 2021). Previous studies have shown that the structure of textbook explanations, examples, and exercises significantly affects students' opportunities to develop mathematical understanding. From this perspective, textbooks can be viewed as institutional representations of mathematical knowledge that shape not only what students learn but also how they are expected to reason, justify, and validate mathematical ideas. Therefore, investigating how exponent concepts are organized within textbooks is essential for understanding the epistemological nature of the learning opportunities provided to students.

In Indonesia, the Electronic School Book (Buku Sekolah Elektronik/BSE) serves as a primary instructional resource for curriculum implementation. As an official textbook widely used in secondary schools, the BSE is expected to provide coherent mathematical explanations and meaningful learning trajectories. However, the quality of learning opportunities offered by textbooks depends not only on the availability of tasks but also on how mathematical knowledge is organized and justified throughout the learning process. Recent studies have emphasized that textbook analysis should move beyond identifying content coverage and instead examine the didactical and epistemological structures embedded within mathematical tasks and explanations (Yunianta et al., 2023; Ismayanti et al., 2025). Such analyses allow researchers to investigate how mathematical knowledge is transformed from scholarly mathematics into teachable school mathematics and how this transformation influences students' opportunities to construct understanding. Consequently, understanding the organization of exponentiation within the BSE requires attention not only to what mathematical content is presented but also to how that content is justified, connected, and made meaningful for learners.

To investigate how mathematical knowledge is organized within instructional materials, this study adopts the Anthropological Theory of the Didactic (ATD), originally developed by Chevallard. ATD conceptualizes mathematical activity through the notion of praxeology, consisting of four interconnected components: types of tasks (T), techniques (τ), technologies (θ), and theories (Θ). Types of tasks and techniques constitute the praxis block, whereas technologies and theories form the logos block that legitimizes mathematical practices. Meaningful mathematical activity emerges from coherent relationships among these four components, as techniques become educationally significant only when supported by explicit explanations and theoretical justifications (Chevallard, 2022). From this perspective, conceptual understanding can be interpreted as learners' ability to connect mathematical techniques with the technological and theoretical knowledge that explains and legitimizes those techniques. Consequently, the relationship between praxis and logos becomes central to understanding how mathematical meaning is constructed within instructional materials. Recent developments in ATD research further emphasize its importance as an epistemological framework for investigating how mathematical knowledge is produced, organized, transmitted, and institutionalized within educational systems (Gascón, 2024; Abou-Hayt, 2026; Zamanabadi & Rafiepour, 2026).

Praxeological analysis has increasingly been applied to investigate mathematical textbooks, classroom practices, learning obstacles, and didactical designs. Previous studies have examined praxeological organizations in exponent equations, number patterns, algebraic operations, derivatives, engineering mathematics, and textbook presentations of various mathematical topics (Wijayanti & Aufa, 2020; Peters, 2022; Jatisunda et al., 2025; Pradipta et al., 2025). Collectively, these studies demonstrate that mathematical knowledge is often presented through rich collections of tasks and techniques, while the technological and theoretical justifications underlying these techniques remain less visible. Similar findings have also been reported in textbook studies, where praxeological structures frequently place greater emphasis on praxis than on logos (Yunianta et al., 2023; Ismayanti et al., 2025). Such imbalances may limit students' opportunities to understand the mathematical meanings underlying procedures and may contribute to the emergence of epistemological obstacles in learning. From an ATD perspective, this pattern raises an important theoretical concern because techniques weakly connected to their technological and theoretical foundations may be learned as procedural routines rather than meaningful mathematical practices.

Despite the growing body of ATD research, an important issue remains insufficiently understood. Existing studies have examined students' misconceptions, learning obstacles, instructional interventions, and various textbook structures related to exponentiation. Similarly, ATD-based investigations have successfully identified praxeological components across different mathematical topics. Nevertheless, limited attention has been paid to how knowledge of exponents is organized across an entire introductory learning trajectory and how relationships among tasks, techniques, technologies, and theories evolve throughout that trajectory. As a result, little is known about how textbook presentations progressively transform contextual meanings into generalized mathematical structures and eventually into institutionalized mathematical knowledge. From an ATD perspective, the central concern is not merely whether praxeological components are present, but whether their organization enables students to connect procedures with the explanations and theoretical principles that justify them. Unlike previous studies that primarily focused on isolated exponent tasks, learning difficulties, instructional interventions, or the identification of individual praxeological components, this study reconstructs the complete praxeological organization of introductory exponent learning. Particular attention is given to how mathematical meaning evolves across successive learning phases and how the relationships among tasks, techniques, technologies, and theories support the transition from meaning construction to formal mathematical practice. In this way, the study contributes not only to textbook research but also to theoretical discussions concerning the epistemological development of school mathematical knowledge within the framework of the Anthropological Theory of the Didactic.

Therefore, this study aims to investigate the praxeological organization of the presentation of exponentiation in the 2021 Indonesian Grade 10 Mathematics Electronic School Book (BSE). Particular attention is given to the relationships among types of tasks, techniques, technologies, and theories contained in Exploration 1.1, Exploration 1.2, and Exercise 1.1. By examining how the balance between praxis and logos is constructed throughout the introductory exponent learning trajectory, this study seeks to contribute to theoretical discussions within ATD concerning the organization and justification of school mathematical knowledge. Furthermore, the study provides empirical evidence regarding how textbook tasks can support the development of conceptual understanding alongside procedural fluency. The findings may inform textbook design by illustrating how mathematical content can be sequenced to facilitate the construction of meaning, the generalization of patterns, and the institutionalization of mathematical knowledge. Consequently, the study contributes to both the advancement of praxeological textbook research and the improvement of mathematics learning resources.

METHODS

Research Design

This study employed a qualitative document analysis design to investigate how exponent concepts are organized and presented within a mathematics textbook through the perspective of the Anthropological Theory of the Didactic (ATD). Qualitative document analysis was considered appropriate because the study focused on examining the organization of mathematical knowledge embedded in curriculum resources and instructional materials rather than evaluating students' learning outcomes or classroom practices (Pepin & Gueudet, 2020; Zeynivandnezhad et al., 2024). Textbooks constitute a central component of curriculum implementation and provide valuable evidence regarding how mathematical knowledge is selected, structured, and communicated to learners (Sunzuma, 2026). Following the ATD perspective, the analysis aimed to identify and interpret the relationships among types of tasks (T), techniques (τ), technologies (θ), and theories (Θ) that collectively constitute textbook praxeologies (Chevallard, 2019). Within this framework, mathematical knowledge is understood as a system of human practices and discourses organized through the interaction between practical activities and their corresponding justifications. Particular attention was therefore given to examining the balance between the praxis block (tasks and techniques) and the logos block (technologies and theories) in order to understand how the textbook organizes opportunities for students to construct, justify, and apply expert knowledge (Chevallard & Bosch, 2020).

Data Source and Unit of Analysis

The primary data source was the 2021 Grade 10 Mathematics Electronic School Book (Buku Sekolah Elektronik/BSE) published by the Indonesian Ministry of Education and authored by Susanto et al. (2021). The analysis focused on the introductory exponent chapter, specifically Exploration 1.1, Exploration 1.2, and Exercise 1.1. These sections were purposively selected because they constitute the initial learning trajectory through which students encounter concepts of exponentiation before progressing to more advanced exponent-related topics. Consequently, they provide an appropriate context for investigating how knowledge of exponents is constructed, generalized, and institutionalized within the textbook. The unit of analysis consisted of individual mathematical tasks. In this study, a task was defined as a mathematically meaningful activity that required students to perform a specific mathematical action to achieve a particular objective. Tasks included exploratory questions, problem-solving prompts, proof activities, and simplification exercises. Each task was analyzed as a potential praxeological unit containing identifiable task types, techniques, technologies, and theories.

Analytical Framework

The study was guided by the Anthropological Theory of the Didactic (ATD) developed by Chevallard (2019). Within ATD, mathematical activity is conceptualized through praxeology, which comprises four interconnected components: type of task (T), technique (τ), technology (θ), and theory (Θ) (Chevallard, 2019). The type of task refers to the mathematical activity required from learners, whereas technique denotes the procedures used to accomplish the task. Technology consists of the explanations and justifications that support the use of particular techniques, while theory refers to the broader mathematical principles that legitimize these explanations (Chevallard & Bosch, 2020). The first two components constitute the praxis block, representing the practical dimension of mathematical activity, whereas technology and theory form the logos block, which provides the rationale and legitimacy for mathematical practices (Chevallard, 2019). From an ATD perspective, meaningful mathematical activity emerges through the articulation of praxis and logos rather than through the execution of procedures alone. This framework, therefore, enables the investigation of both procedural and conceptual dimensions of textbook presentations by examining how tasks, techniques, technologies, and theories are organized within instructional materials. To ensure analytical consistency, each praxeological component was identified using the analytical criteria presented in Table 1.

Table 1. *Operational Criteria for Praxeological Analysis*

Component	Operational Definition	Analytical Indicator
Type of Task (T)	Mathematical activity proposed to students	What students are asked to do
Technique (τ)	Procedure used to complete the task	How the task is solved
Technology (θ)	Explanation justifying the technique	Why the technique works
Theory (Θ)	Formal mathematical principle underlying the technology	Mathematical concepts or laws supporting the explanation

Data Collection

Data were collected through systematic document analysis, a method commonly employed to examine the organization and representation of knowledge within educational materials (Pepin & Gueudet, 2020). The selected textbook sections were repeatedly examined to identify all mathematical tasks involving exponentiation. Each task was extracted and recorded in a praxeological analysis matrix. For each identified task, information on the required mathematical action, solution procedures, explanatory statements, and formal mathematical principles was documented. This process enabled the researchers to establish preliminary relationships among task types, techniques, technologies, and theories before conducting a deeper analysis of the resulting praxeological organization.

Data Analysis

Data analysis was conducted through a praxeological analysis procedure grounded in the Anthropological Theory of the Didactic (ATD). The analysis aimed to identify how exponent knowledge was organized through the relationships among types of tasks (T), techniques (τ), technologies (θ), and theories (Θ). Following previous ATD studies on textbook analysis, the analytical process moved progressively from identifying mathematical activities proposed to students toward examining the extent to which these activities were supported by explicit technological and theoretical justifications. This procedure enabled the reconstruction of local praxeological organizations and facilitated an examination of the balance between praxis and logos in the textbook. The stages of analysis are summarized in Table 2.

Table 2. *Analytical Procedure for Reconstructing the Praxeological Organization of Exponent Concepts*

Stage	Description
Task Identification	All mathematical tasks in Exploration 1.1, Exploration 1.2, and Exercise 1.1 were identified and classified by their mathematical objectives and expected student activities.
Technique Identification	The procedures required to accomplish each task were identified and described. Similar procedures were subsequently grouped into common technique categories.
Technology Identification	Explanations, rationales, and mathematical justifications accompanying each technique were examined to identify the technological component of the praxeology.
Theory Identification	The broader mathematical principles underlying the identified technologies were determined, including definitions, exponent properties, and formal mathematical structures that legitimize the techniques.

Construction of Praxeological Organization	Relationships among task types, techniques, technologies, and theories were organized into local praxeological structures. These structures were compared across Exploration 1.1, Exploration 1.2, and Exercise 1.1 to identify patterns in the presentation of exponent concepts.
Analysis of Praxis–Logos Balance	The resulting praxeological structures were examined to determine the relative prominence of the praxis block (T and τ) and the logos block (θ and Θ). Particular attention was given to the extent to which techniques were supported by explicit technological explanations and theoretical justifications.

Trustworthiness

Several strategies were employed to enhance the trustworthiness of the findings. First, iterative readings of the textbook were conducted to ensure consistency in task identification and categorization, a procedure commonly recommended in document-based qualitative research (Bowen, 2009). Second, all coding decisions were continuously compared with the theoretical definitions of praxeological components proposed within ATD literature to maintain theoretical consistency. Third, peer debriefing sessions were conducted among mathematics education researchers familiar with ATD to discuss coding decisions and resolve interpretative differences, thereby strengthening the credibility of the analysis (Lincoln & Guba, 1994). Finally, all identified praxeological structures were re-examined against the original textbook content to ensure that interpretations remained grounded in the data and theoretically coherent. These procedures contributed to the dependability and credibility of the reconstructed praxeological organization.

RESULT

Overview of the Praxeological Organization of Exponent Learning

The analysis focused on Exploration 1.1, Exploration 1.2, and Exercise 1.1 of the exponent chapter in the 2021 Grade 10 Mathematics Electronic School Book (BSE). Across these sections, concepts of exponents are introduced through a sequence of learning activities that progress from contextual exploration to increasingly formal mathematical tasks. Exploration 1.1 presents exponentiation through a repeated-growth context, whereas Exploration 1.2 engages students in identifying regularities in exponent operations and in formulating exponent properties. Exercise 1.1 subsequently provides opportunities to apply these properties in proofs, equation solving, and expression simplification. The identified tasks were reconstructed as local praxeological organizations consisting of types of tasks (T), techniques (τ), technologies (θ), and theories (Θ). The resulting analysis reveals how knowledge of exponents is progressively developed throughout the chapter and how the relationships among praxeological components evolve over the learning trajectory. The following sections present the reconstructed praxeologies for each part of the chapter before examining the overall relationship between the praxis and logos blocks.

Exploration 1.1: Constructing Exponentiation as Repeated Growth

Exploration 1.1 introduces exponentiation through a contextual situation involving virus transmission. Rather than presenting exponent notation directly, the textbook invites students to observe how quantities increase across successive phases of transmission. This choice positions exponentiation as a representation of a multiplicative growth process before it is introduced as a symbolic mathematical object.

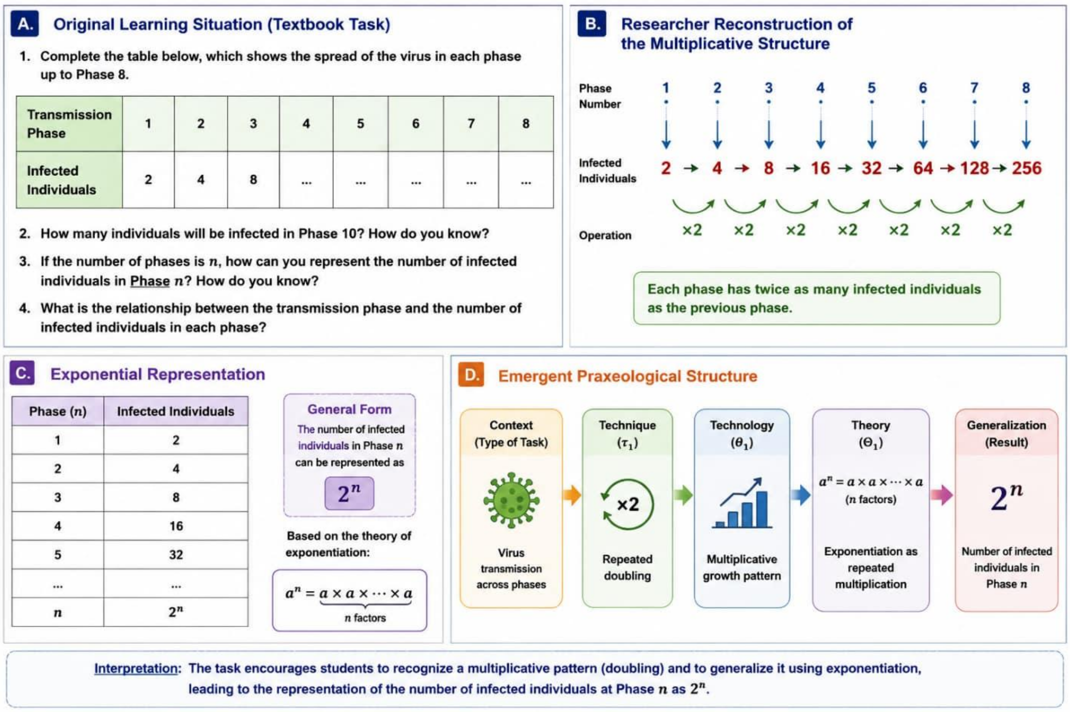

Figure 1. *Virus transmission activity in Exploration 1.1*

Table 1. *Praxeological Analysis of Task 1 (T_1)*

Praxeological Component	Description
Type of Task (T_1)	Generalizing the number of infected individuals across transmission phases and representing the relationship between phase number and quantity.
Technique (τ_1)	Determining the number of infected individuals by repeatedly doubling the quantity from the previous phase and extending the resulting numerical pattern to higher phases.
Technology (θ_1)	The relationship between successive phases is multiplicative rather than additive because each phase contains twice as many infected individuals as the preceding phase. This regularity yields a repeated-multiplication structure that can be expressed symbolically in terms of powers of two, enabling prediction of quantities at distant phases without enumerating every intermediate step.
Theory (Θ_1)	Exponentiation as repeated multiplication: a^ndenotes the product of $n$identical factors of a, that is, $a^n = a \times a \times \cdots \times a$ (n factors) Accordingly, the sequence $2,4,8,16, \ldots = 2^1, 2^2, 2^3, 2^4, \ldots$ establishes a general relationship between the transmission phase and the number of infected individuals, where the quantity at the phase $n$can be represented as 2^n.

The praxeology reconstructed from Task 1 reveals that exponentiation is initially organized around the idea of repeated growth. The task requires students to extend a sequence generated by successive doubling, while the identified technique involves repeatedly multiplying by a constant factor. Through this technique, students engage with a multiplicative structure that gradually exceeds the practicality of repeated calculation. The technology associated with this technique explains that the quantity in each phase is obtained by doubling the quantity from the previous phase. Consequently, exponent notation emerges as a more efficient way of representing growth patterns. The corresponding theory is grounded in the definition of exponentiation as repeated multiplication. From a praxeological perspective, this task performs an important didactical function. Exponentiation is not introduced as an arbitrary symbolic convention but as a mathematical response to a recurring multiplicative phenomenon. The relationship between technique, technology, and theory, therefore, supports the construction of meaning before the introduction of formal exponent properties. This organization creates opportunities for students to connect contextual reasoning with symbolic representation.

Exploration 1.2: From Operational Patterns to Generalized Exponent Rules

Following the construction of exponentiation as repeated growth, Exploration 1.2 redirects attention toward operations involving exponential expressions. The focus shifts from interpreting quantities to identifying structural relationships among exponents.

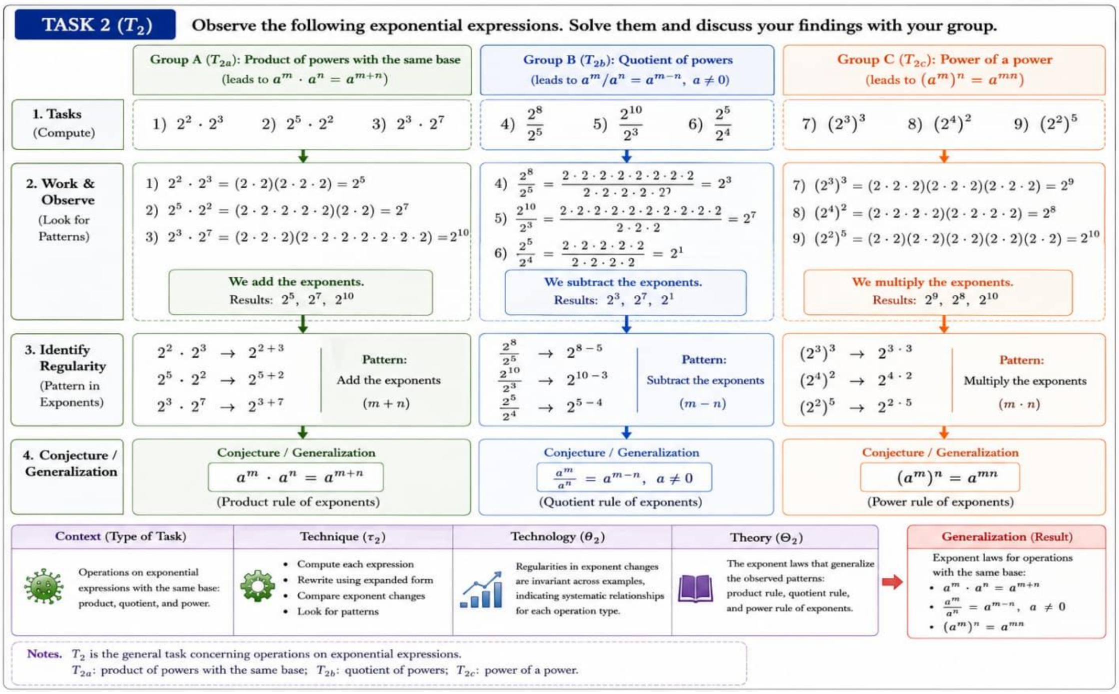

Figure 2. *Exponent operations activity Task 2 (T_2)*

Table 2. *Praxeological Analysis of Task 2 (T_2)*

Praxeological Component	Description
Type of Task (T_2)	Determining the numerical results of operations involving exponential expressions and identifying regularities that emerge from multiplication, quotient, and repeated exponentiation tasks in order to formulate general exponent rules.
Technique (τ_2)	Expanding exponential expressions into repeated factors, performing multiplication or division operations, simplifying the resulting expressions, and rewriting the results in exponential form to compare outcomes across tasks and identify recurring patterns.

Technology (θ_2)	Exponential notation represents repeated multiplication of identical factors. By interpreting powers as collections of equal factors, multiplication combines collections, division removes common factors, and repeated exponentiation replicates an entire collection multiple times. The regularities observed across these operations provide a justification for generalizing numerical results into symbolic exponent rules without the need for complete factor expansion.
Theory (Θ_2)	The algebraic system of exponent laws is derived from the definition of exponents as repeated multiplication. This system enables the generalization of observed numerical patterns into symbolic relationships, including the product rule $a^m \times a^n = a^{m+n}$, the quotient rule $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$, and the power rule $(a^m)^n = a^{mn}$.

Task 2 introduces operations involving exponential expressions and invites students to investigate regularities emerging from multiplication, division, and repeated exponentiation. The technique consists of evaluating each expression, rewriting the results in exponential form, and comparing the resulting exponents across tasks. Unlike the previous exploration activities, exponentiation is treated here as an object of algebraic manipulation rather than as a contextual representation. The associated technology is grounded in the interpretation of exponents as repeated multiplication of identical factors: multiplication combines collections of factors, division removes common factors, and repeated exponentiation replicates an entire collection multiple times. These interpretations justify the observed regularities in exponent values and support the transition from numerical calculations to symbolic generalizations. The resulting exponent laws—the product rule, quotient rule, and power rule—constitute the theoretical component that institutionalizes these regularities within a coherent algebraic system.

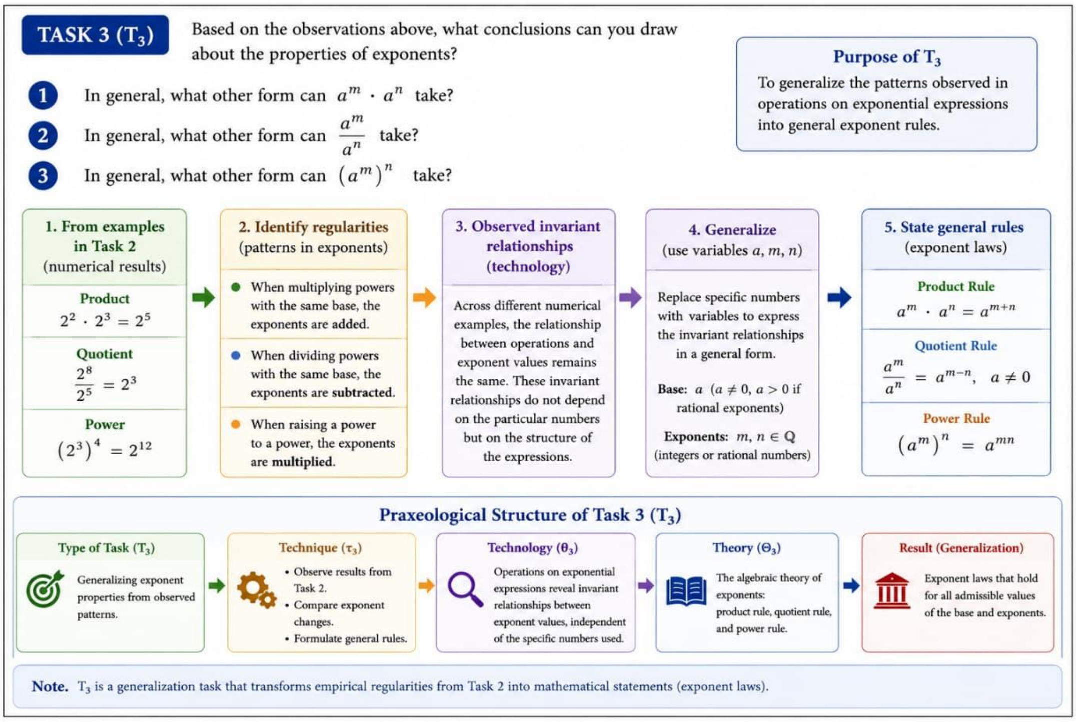

Figure 3. *Generalization of exponent properties*

Table 3. *Praxeological Analysis of Task 3 (T_3)*

Praxeological Component	Description
Type of Task (T_3)	Generalizing exponent properties by formulating symbolic rules that account for the regularities observed in multiplication, division, and repeated exponentiation of powers.
Technique (τ_3)	Comparing the results obtained in previous tasks, identifying invariant relationships between exponent values and operations, and expressing these relationships using algebraic notation involving variables.
Technology (θ_3)	The numerical patterns observed across multiple examples suggest stable relationships that are independent of particular numerical values. By replacing specific numbers with variables, students recognize that operations with exponents follow general structural relationships. This transition from numerical regularities to symbolic expressions justifies formulating exponent laws as universally applicable rules rather than as isolated computational shortcuts.
Theory (Θ_3)	The algebraic theory of exponents formalizes the regularities identified through previous tasks. The resulting exponent laws constitute a coherent system governing operations on powers: $a^m \times a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$, and $(a^m)^n = a^{mn}$.

Task 3 represents a shift from empirical pattern recognition to algebraic generalization. Whereas the previous tasks focused on evaluating particular exponential expressions, students are now required to formulate general rules using symbolic notation. The technique involves identifying invariant relationships among previously computed examples and expressing these regularities in terms of variables. The associated technology rests on the idea that the observed patterns are independent of specific numerical values and therefore reflect underlying structural properties of exponentiation. Through this process, exponent laws become institutionalized as theoretical objects that generalize and justify the regularities identified in earlier tasks.

Exercise 1.1: Institutionalization of Exponent Knowledge

Exercise 1.1 represents the institutionalization phase of exponent learning. The exponent properties previously developed through exploration now serve as mathematical tools for justifying statements, solving equations, and simplifying expressions.

Justifying Exponent Properties

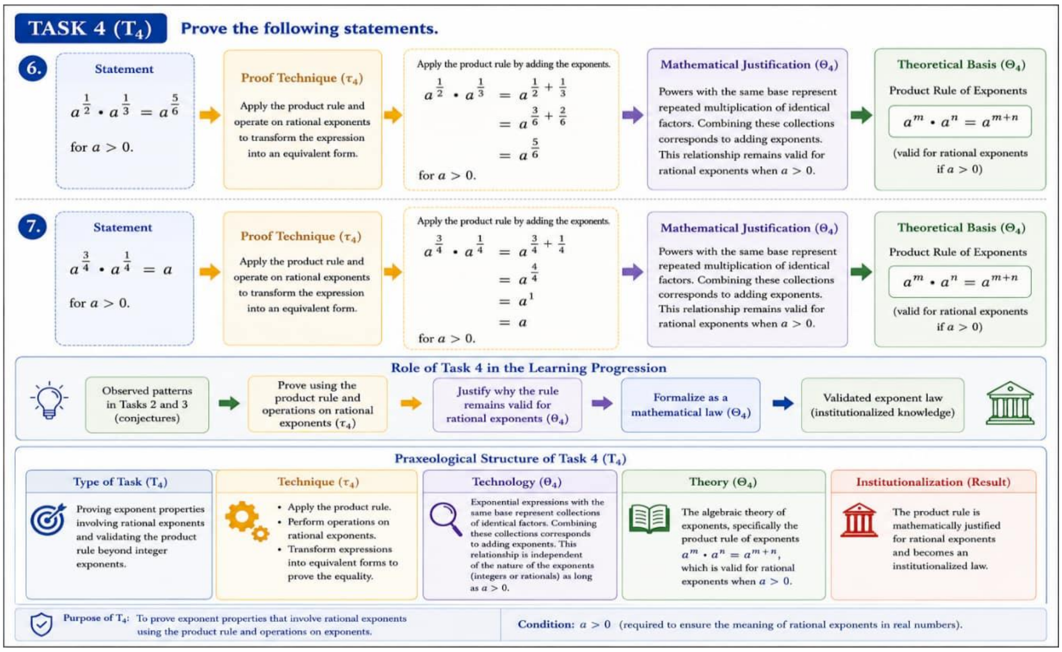

Figure 4. *Justifying Exponent Properties involving rational exponents*

Table 4. *Praxeological Analysis of Task 4 (T_4)*

Praxeological Component	Description
Type of Task (T_4)	Proving exponent properties involving rational exponents and validating the extension of exponent laws beyond integer exponents.
Technique (τ_4)	Applying the product rule of exponents, performing operations on rational exponents, and transforming the given expressions into equivalent forms to demonstrate the validity of the stated equalities.
Technology (θ_4)	Exponential expressions with the same base represent collections of identical factors whose combination corresponds to adding exponents. This structural relationship is independent of whether the exponents are integers or rational numbers. Consequently, the product rule remains valid for rational exponents, providing a mathematical justification for the transformations used in the proof.
Theory (Θ_4)	The algebraic theory of exponents, particularly the product rule $a^m \times a^n = a^{m+n}$, which applies to rational exponents under the appropriate domain conditions ($a > 0$).

Task 4 represents a significant shift from pattern generalization to mathematical justification. Whereas the previous tasks encouraged students to formulate exponent laws from observed regularities, this task requires them to validate the applicability of those laws to rational exponents. The technique involves transforming the given expressions by applying the product rule and performing operations on rational exponents to establish the stated equalities. The associated technology is based on interpreting exponential expressions as products of identical factors. When powers with the same base are multiplied, the bases are kept the same and the exponents are added, which corresponds to multiplying the exponents. Importantly, this structural relationship does not depend on the exponents being integers; it remains valid when exponents are rational numbers. This provides a justification for extending previously established exponent laws to a broader numerical domain. The theoretical component is the algebraic theory of exponents, specifically the product rule, a. to them times a. to the n equals a to the $a^m \times a^n = a^{m+n}, m, n \in Q, a > 0$. Within this task, the exponent law functions not merely as a computational shortcut but as a mathematical statement whose validity must be demonstrated. Consequently, Task 4 marks the beginning of the institutionalization process in which exponent laws become objects of mathematical validation and justification rather than empirical regularities derived from numerical examples.

Solving Exponential Equations

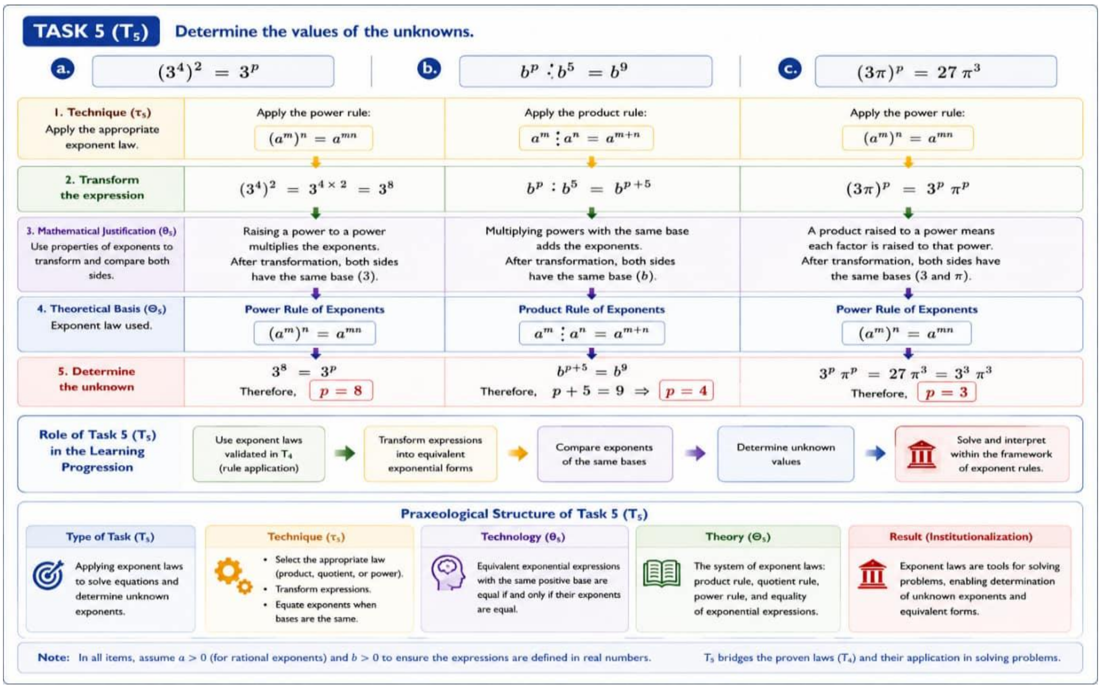

Figure 5. *An exponential equation requires the application of the power rule to determine an unknown variable.*

Table 5. *Praxeological Analysis of Task 5 (T_5)*

Praxeological Component	Description
Type of Task (T_5)	Determining the value of an unknown variable in an exponential equation using the power rule of exponents.
Technique (τ_5)	Applying the power rule of exponents to simplify the left-hand side into a single exponential expression and subsequently determining the value of $p$by equating the exponents on both sides of the equation.
Technology (θ_5)	Repeated exponentiation can be expressed as a single exponential form through the multiplication of exponents. When exponential expressions are rewritten with the same base, the equality of the expressions implies the equality of their exponents. This relationship allows the unknown variable to be determined by comparing exponents rather than by direct numerical computation.

Theory (Θ_5)	Power rule of exponents: $(a^m)^n = a^{mn}$; one-to-one property of exponential functions: if $a^x = a^y$, then $x = y$, for $a > 0$and $a \neq 1$.

Task 5 focuses on determining the value of an unknown variable in an exponential equation. Unlike Task 4, which requires students to justify the validity of an exponent property, this task requires them to use the laws of exponents as mathematical tools to solve an equation. The type of task, therefore, shifts from proof-oriented activity to algebraic problem solving. The identified technique simplifies repeated exponentiation by applying the power of a power rule. Through this transformation, the exponential expression on the left-hand side is rewritten as a single exponential. Once both sides of the equation are expressed with the same base, the value of the unknown variable can be obtained by equating the corresponding exponents. The associated technology explains why this procedure is mathematically valid. Repeated exponentiation corresponds to multiplying exponents, allowing nested exponential expressions to be simplified to a single exponential form. Furthermore, when two exponential expressions share an identical base, equality between the expressions implies equality between their exponents. This relationship allows students to determine the unknown variable by comparing exponents rather than by direct numerical evaluation. The corresponding theory is supported by the power rule of exponents, $(a^m)^n = a^{mn}$, together with the one-to-one property of exponential functions. Consequently, the resulting praxeology illustrates how exponent properties become operational tools for solving mathematical problems. From an ATD perspective, the task contributes to the institutionalization of exponent knowledge by transforming previously generalized exponent laws into techniques for solving exponential equations.

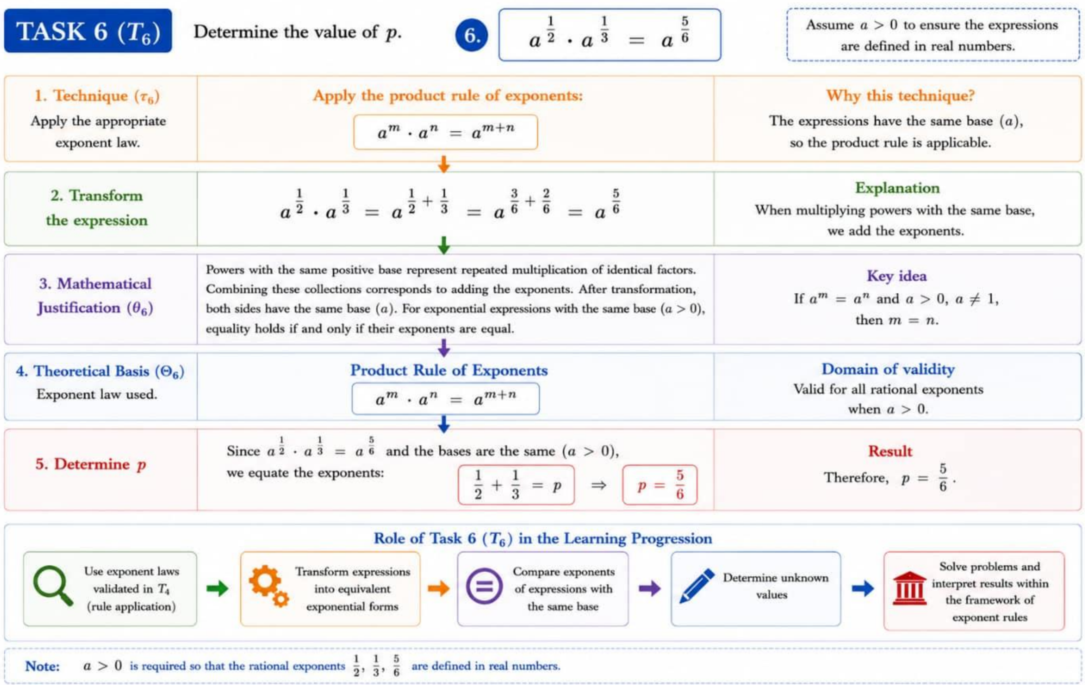

Figure 6. *Task for determining an unknown variable in an exponential equation using the product rule of exponents.*

Table 6. *Praxeological Analysis of Task 6 (T_6)*

Praxeological Component	Description
Type of Task (T_6)	Determining the value of an unknown variable in an exponential equation using the product rule of exponents.
Technique (τ_6)	Applying the product rule of exponents to combine exponential expressions with the same base, simplifying the equation, and subsequently determining the value of $p$by equating the exponents on both sides.
Technology (θ_6)	When exponential expressions with the same base are multiplied, the total number of identical factors increases, resulting in the addition of exponents. After simplification, equivalent exponential expressions with the same base can be compared directly through their exponents, allowing the unknown variable to be determined.
Theory (Θ_6)	Product rule of exponents: $a^m \times a^n = a^{m+n}$; one-to-one property of exponential functions: if $a^x = a^y$, then $x = y$, for $a > 0$and $a \neq 1$.

Task 6 focuses on determining the value of an unknown variable through the application of the product rule of exponents. Unlike Task 5, which relies on the power rule, this task requires students to simplify exponential expressions by combining terms that share the same base. The type of task, therefore, emphasizes the use of exponent multiplication properties within an algebraic equation. The identified technique involves applying the product rule to combine exponential expressions and rewriting the equation in a simplified form. Once equivalent bases are obtained on both sides of the equation, the value of the unknown variable can be determined by comparing the corresponding exponents. The associated technology explains that multiplying exponential expressions with identical bases increases the total number of repeated factors, thereby adding exponents. This relationship enables students to transform complex expressions into equivalent forms that are easier to analyze. The corresponding theory is the product rule of exponents, together with the one-to-one property of exponential functions. Consequently, the resulting praxeology demonstrates how exponent multiplication properties function as operational tools for solving exponential equations. From an ATD perspective, the task contributes to the institutionalization of exponent laws by emphasizing their application in algebraic problem-solving contexts.

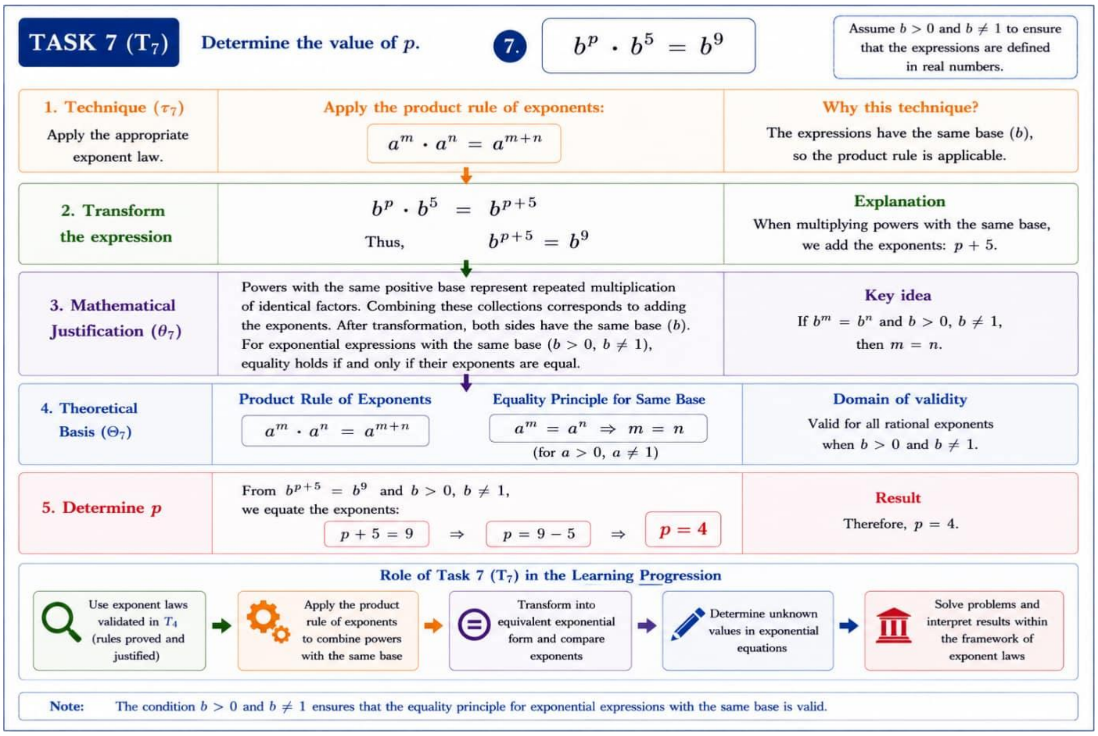

Figure 7. *Task for determining an unknown variable in an exponential equation using the product rule of exponents.*

Table 7. *Praxeological Analysis of Task 7 (T_7)*

Praxeological Component	Description
Type of Task (T_7)	Determining the value of an unknown variable by establishing equivalent exponential expressions with a common base.
Technique (τ_7)	Rewriting one side of the equation as an exponential expression with the same base and subsequently applying exponent properties to determine the value of p.
Technology (θ_7)	Rewriting exponential expressions into equivalent forms with a common base allows direct comparison between the two sides of the equation. Once the bases are identical, the relationship between the exponents can be used to determine the unknown variable.
Theory (Θ_7)	Power of a product property: $(ab)^n = a^n b^n$; one-to-one property of exponential functions: if $a^x = a^y$, then $x = y$, for $a > 0$ and $a \neq 1$.

Task 7 introduces a more sophisticated form of exponential equation solving by requiring students to establish equivalent exponential expressions before determining the value of the unknown variable. The task differs from Tasks 5 and 6 because the solution cannot be obtained directly through a single exponent property. The identified technique involves transforming one side of the equation into an equivalent exponential form with a common base. This transformation enables the two expressions to be compared directly and allows the value of the variable to be determined by comparing exponents. The associated technology emphasizes the importance of base equivalence in solving exponential equations. Equivalent bases provide a common mathematical structure that enables systematic examination of relationships between exponents. The corresponding theory is based on the power property of a product and the one-to-one property of exponential functions. The resulting praxeology highlights a higher level of procedural flexibility than that observed in Tasks 5 and 6. Students must not only apply exponent properties but also select appropriate transformations that facilitate the solution process. Consequently, the task reinforces the role of exponent laws as strategic tools for algebraic reasoning.

Simplifying Exponential Expressions

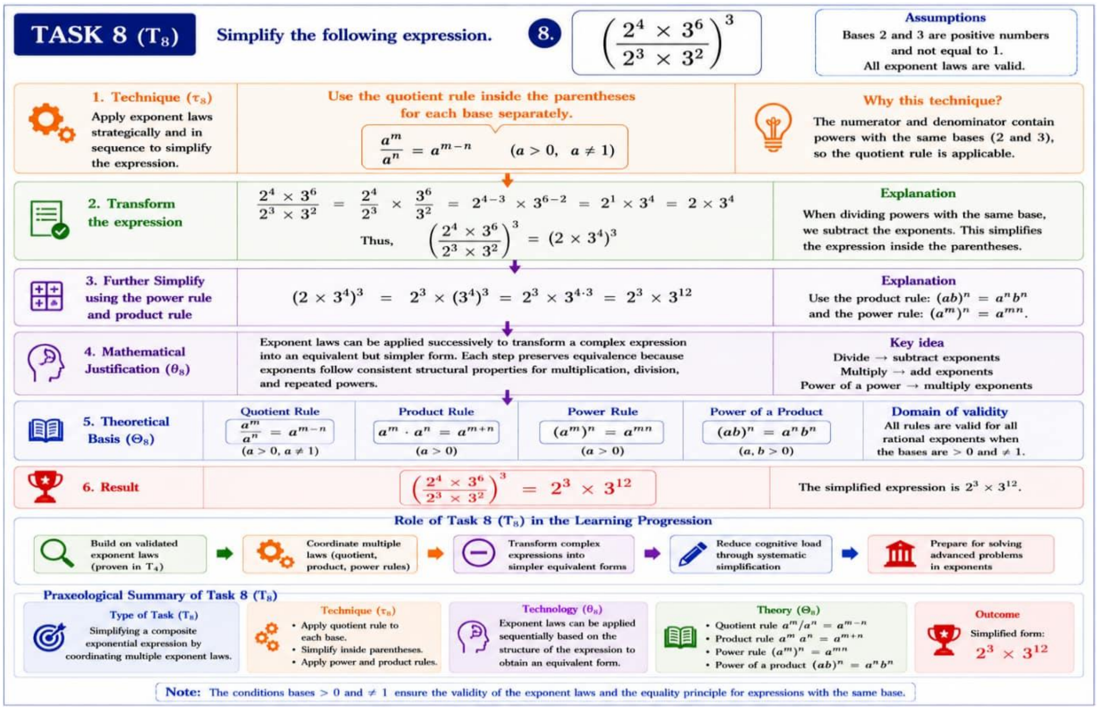

Figure 8. *Task involving the simplification of an exponential expression using quotient and power properties.*

Table 8. *Praxeological Analysis of Task 8 (T_8)*

Praxeological Component	Description
Type of Task (T_8)	Simplifying exponential expressions involving quotient and power properties.
Technique (τ_8)	Applying the quotient and power properties of exponents sequentially to transform the expression into a simpler exponential form.
Technology (θ_8)	Exponent operations involving division and exponentiation follow consistent algebraic rules that preserve equivalence between expressions. These relationships allow complex exponential expressions to be simplified without changing their mathematical value.
Theory (Θ_8)	Quotient rule of exponents: $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$; power of a product property: $(ab)^n = a^n b^n$.

Task 8 focuses on simplifying exponential expressions through the coordinated application of multiple properties of exponents. Unlike the preceding tasks, which involve determining unknown variables, this task requires students to transform an expression into a more concise equivalent form. The identified technique involves applying quotient and power properties in sequence to simplify the given expression. The associated technology states that exponent operations obey consistent algebraic relationships that preserve equivalence under transformations. This allows students to simplify expressions systematically without altering their mathematical meaning. The corresponding theories consist of the quotient rule and the power of a product property. Consequently, the resulting praxeology illustrates how exponent laws serve as tools for symbolic manipulation and simplification of expressions.

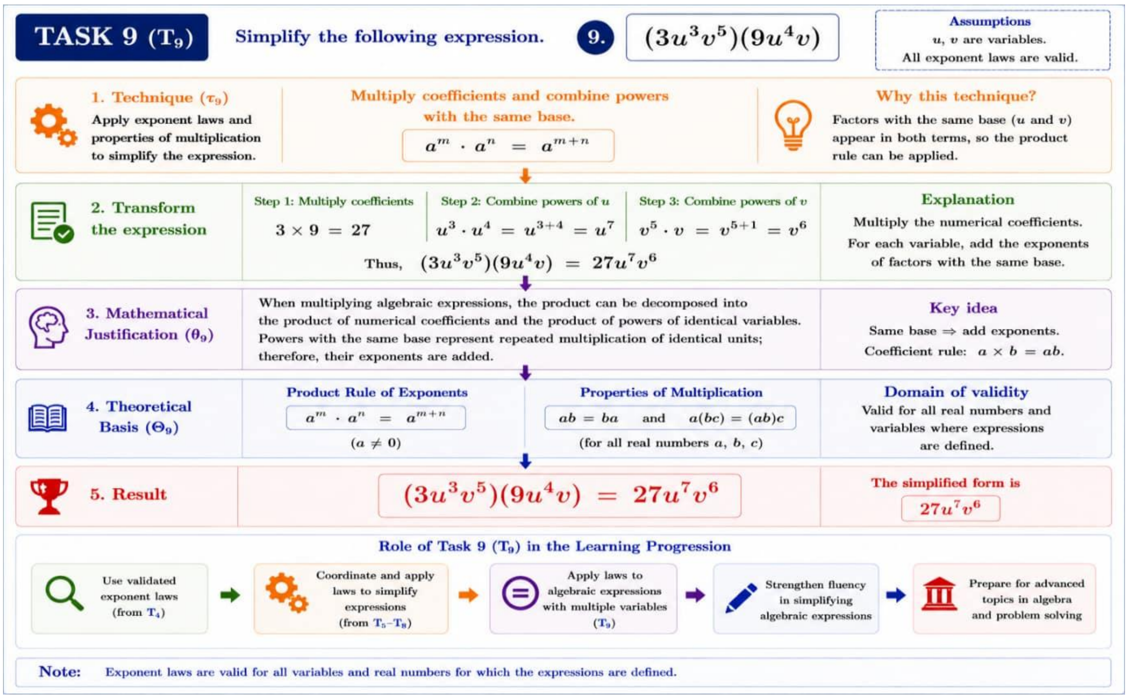

Figure 9. *Task involving the simplification of an exponential expression using the product rule of exponents.*

Table 9. *Praxeological Analysis of Task 9 (T_9)*

Praxeological Component	Description
Type of Task (T_9)	Simplifying exponential expressions by combining terms with identical bases.
Technique (τ_9)	Applying the product rule of exponents to combine coefficients and variables that share the same base.
Technology (θ_9)	Multiplication of exponential expressions with identical bases results in the accumulation of repeated factors, allowing exponents to be added and expressions to be rewritten in a more compact form.
Theory (Θ_9)	Product rule of exponents: $a^m \times a^n = a^{m+n}$.

Task 9 requires students to simplify exponential expressions using the product rule of exponents. The task emphasizes recognizing common bases and using exponent addition to combine equivalent factors. The identified technique involves grouping terms with identical bases and applying the product rule to obtain a simplified expression. The associated technology explains that multiplication combines repeated factors, thereby increasing the exponent associated with a given base. This relationship provides the mathematical justification for exponent addition. The corresponding theory is represented by the formal product rule of exponents. As a result, the praxeology reinforces students' understanding of exponent multiplication as a mechanism for simplifying symbolic expressions and consolidating equivalent factors.

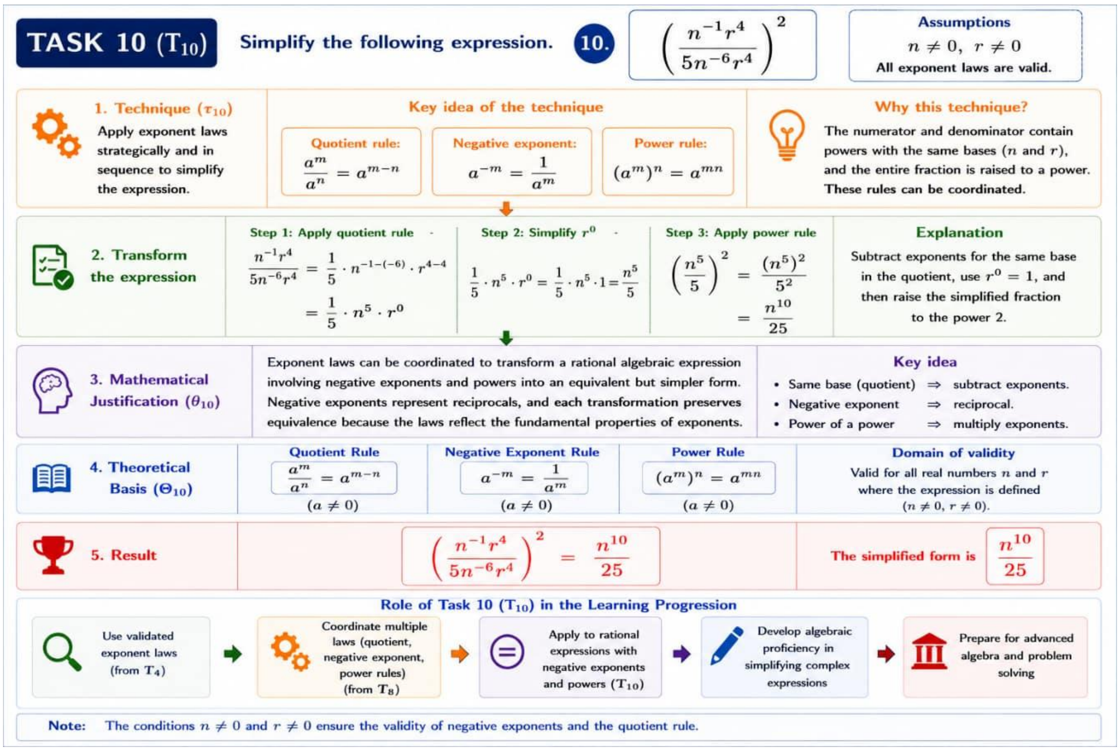

Figure 10. *Task involving the simplification of an exponential expression through the coordinated application of multiple exponent properties.*

Table 10. *Praxeological Analysis of Task 10 (T_{10})*

Praxeological Component	Description
Type of Task (T_{10})	Simplifying exponential expressions through the coordinated application of multiple exponent properties.
Technique (τ_{10})	Applying quotient rules, quotient power properties, and the zero-exponent property to transform the expression into a simpler equivalent form.
Technology (θ_{10})	Division of exponential expressions with identical bases reduces the number of repeated factors, resulting in exponent subtraction. Additional transformations involving quotient powers and zero exponents preserve equivalence while producing a more concise representation of the expression.
Theory (Θ_{10})	Quotient rule of exponents: $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$; quotient power rule: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$; zero-exponent property: $a^0 = 1$, $a \neq 0$.

Task 10 is the most complex simplification activity in the chapter because it requires the coordinated use of several properties of exponents within a single solution process. The task focuses on transforming a complex exponential expression into an equivalent but simpler form. The identified technique involves applying quotient rules, quotient power properties, and the zero-exponent property in a systematic sequence. The associated technology explains how these properties of exponents preserve equivalence during symbolic transformations while reducing the complexity of the expression. The corresponding theories consist of the quotient rule, quotient power rule, and zero-exponent property. Consequently, the resulting praxeology illustrates the consolidation of exponent knowledge, where multiple exponent laws are employed simultaneously as mathematical tools for symbolic simplification. From an ATD perspective, this task represents the culmination of the institutionalization process, in which exponent properties function as an integrated system of procedures rather than isolated mathematical rules.

Global Praxeological Organization of Exponent Learning

Beyond the local praxeologies reconstructed from individual tasks, the chapter exhibits a coherent global praxeological organization. The sequence of tasks shows how knowledge of exponents develops from contextual interpretation to formal mathematical practice. The learning trajectory begins with the construction of meaning through repeated growth, progresses toward the identification and generalization of exponent rules, and culminates in the institutionalization of exponent properties as tools for mathematical reasoning and symbolic manipulation. Figure 11 summarizes the overall organization of exponent learning across the analyzed sections.

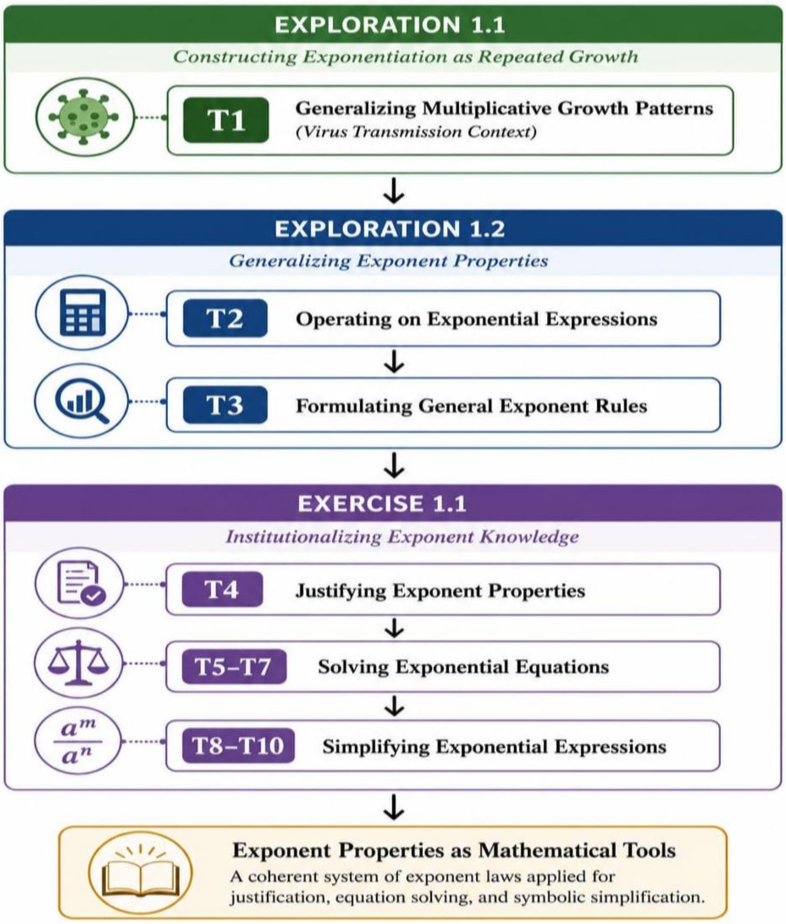

Figure 11. *Global Praxeological Organization of Exponent Learning*

The global organization presented in Figure 11 demonstrates a gradual shift in the role of exponentiation throughout the chapter. In Exploration 1.1, exponentiation emerges as a representation of repeated multiplicative growth. Exploration 1.2 subsequently transforms this meaning into a system of generalized relationships by identifying operational regularities. Finally, Exercise 1.1 institutionalizes these relationships by positioning exponent properties as mathematical tools for justification, equation solving, and symbolic simplification. The resulting trajectory reflects a progression from meaning construction to formal mathematical practice.

Balance Between Praxis and Logos

The reconstructed praxeologies reveal differences in the relative visibility of the praxis and logos blocks across the chapter. While all tasks involve both components, the emphasis placed on techniques and their justifications varies throughout the learning trajectory. Table 11 summarizes the relative prominence of praxis and logos in each task.

Table 11. Relative Visibility of Praxis and Logos Across Tasks

Task	Main Mathematical Focus	Praxis (T–τ)	Logos (θ–Θ)
T_1	Repeated growth and exponent meaning	Strong	Moderate
T_2	Exponent operations	Strong	Moderate
T_3	Generalization of exponent rules	Moderate	Strong
T_4	Justification of exponent properties	Moderate	Strong
T_5	Solving exponential equations	Strong	Weak
T_6	Solving exponential equations	Strong	Weak
T_7	Solving exponential equations	Strong	Weak
T_8	Simplifying exponential expressions	Strong	Weak
T_9	Simplifying exponential expressions	Strong	Weak
T_{10}	Simplifying exponential expressions	Strong	Weak

Table 11 indicates that the relationship between praxis and logos changes throughout the chapter. In Exploration 1.1 and Exploration 1.2, techniques are closely connected to conceptual explanations and theoretical justifications. Students are encouraged not only to perform procedures but also to recognize the multiplicative structure underlying exponentiation and the mathematical principles governing exponent operations. A different pattern emerges in Exercise 1.1. Although properties of exponents continue to provide the theoretical foundation for the tasks, the primary emphasis shifts toward applying procedures. Tasks 5–10 primarily require students to apply the laws of exponents to solve equations and simplify expressions. As a result, opportunities to perform techniques become increasingly prominent, whereas opportunities to further elaborate technological explanations become less visible. From an ATD perspective, the chapter demonstrates a progressive strengthening of the praxis block during the institutionalization phase. Exponent knowledge is ultimately organized as a system of techniques supported by a relatively stable theoretical structure. While this organization promotes procedural fluency and operational competence, it offers comparatively limited opportunities for students to revisit and further develop the conceptual justifications underlying operations with exponents. This imbalance suggests that the institutionalization of expert knowledge is achieved primarily through the repeated application of established techniques rather than through sustained engagement with the technologies and theories that legitimize them.

DISCUSSION

The reconstructed praxeological organization reveals a coherent learning trajectory through which expert knowledge develops from contextual interpretation to formal mathematical activity. This progression is consistent with broader perspectives on learning trajectories, which emphasize the gradual refinement of mathematical ideas through increasingly sophisticated forms of reasoning rather than the direct transmission of formal knowledge (Weber et al., 2015). The organization described in the textbook demonstrates how students are first invited to interpret a contextual situation, then to recognize regularities, and eventually to engage with formal mathematical structures. Such a progression reflects the view that mathematical knowledge emerges through successive stages of conceptual development in which meanings are reorganized and formalized over time (Ernest, 2003). Rather than presenting exponentiation as a predefined symbolic system, the chapter constructs a pathway for students to progressively transform contextual experiences into generalized mathematical knowledge and formal mathematical practice.

A notable feature of the chapter is the introduction of exponentiation through a multiplicative growth context. The virus-transmission task positions repeated doubling as the central mathematical activity from which exponent representation emerges. This organization aligns with research showing that students' understanding of exponentiation develops more meaningfully when exponential notation is connected to multiplicative structures rather than introduced as a set of symbolic rules (Confrey, 1991; Confrey & Smith, 1994). The task requires students to coordinate quantities across successive phases and recognize the invariant multiplicative relationship underlying the sequence. Such reasoning reflects the development of multiplicative units, which constitute an essential foundation for understanding exponential growth (Hackenberg, 2010). Consequently, exponentiation is presented not merely as repeated multiplication but as a mathematical representation of a recurring multiplicative process, thereby providing a conceptual basis for later formalizations of the properties of exponents.

The progression observed in Exploration 1.2 demonstrates how properties of exponents are constructed by identifying and generalizing operational regularities. Rather than presenting exponent laws directly, the textbook organizes tasks that require students to examine relationships among numerical results and formulate general rules from observed patterns. This approach reflects broader perspectives on mathematical generalization that regard pattern recognition as a mechanism by which learners move from particular cases to generalized structures (Amit & Neria, 2008). The resulting exponent rules emerge from a process of abstraction in which regularities are identified, coordinated, and expressed symbolically. Such an organization is consistent with findings that mathematical structures become more meaningful when students actively construct generalizations rather than receive them as completed formulations (Rivera, 2018; Carraher et al., 2008). Consequently, exponent laws appear as products of mathematical reasoning rather than as isolated procedural conventions.

Exercise 1.1 represents a shift in the didactical function of exponent properties. Once the exponent rules have been established, they are used as mathematical tools for justification, equation solving, and symbolic simplification. This transition reflects a process of institutionalization in which previously constructed knowledge becomes stabilized and available for subsequent mathematical activity (Bosch et al., 2020). The reconstructed praxeologies show that exponent properties are no longer treated primarily as objects of investigation but as resources for accomplishing increasingly formal tasks. Such an organization corresponds to descriptions of institutionalized mathematical knowledge in which techniques become established through repeated use within a coherent body of practice (Bosch, 2015). As a result, the chapter transforms exponent laws from conceptual objects to be constructed into operational instruments that support algebraic reasoning and symbolic manipulation.

The analysis further reveals a gradual shift in the balance between the praxis and logos blocks throughout the chapter. In the exploratory sections, techniques are closely connected to technological explanations and theoretical principles. Students are encouraged to examine why procedures work and how relationships among exponents emerge from multiplicative structures. However, this balance shifts during the institutionalization phase, in which procedural activity becomes increasingly prominent. Similar observations have been reported in ATD-based studies, which describe mathematical organizations as dynamic configurations in which the visibility of technologies and theories may vary according to the didactical objectives of a task (Bosch et al., 2020; Pansell, 2023). The resulting organization suggests that expert knowledge is progressively structured around techniques whose legitimacy is supported by a relatively stable theoretical foundation. Consequently, the praxis block becomes more visible than the logos block during the later stages of learning.

Strengthening praxis has important implications for students' mathematical learning. The chapter provides ample opportunities to apply the properties of exponents across a range of tasks, thereby supporting procedural fluency and operational competence. At the same time, the comparatively limited attention to technological explanations may reduce students' opportunities to revisit the conceptual foundations underlying exponent operations. This observation resonates with arguments that conceptual understanding and procedural fluency should not be treated as competing forms of knowledge but as mutually reinforcing dimensions of mathematical proficiency (Kieran, 2013). Procedural competence becomes more robust when supported by conceptual justification, whereas conceptual understanding is strengthened through meaningful procedural activity (Baroody, 2013). The reconstructed organization, therefore, appears particularly effective in promoting procedural mastery, although additional opportunities for technological reflection could further strengthen students' conceptual understanding of exponentiation.

Viewed as a whole, the chapter presents exponent learning as a process of epistemological development in which mathematical knowledge evolves from contextual interpretation to formalized practice. The reconstructed global praxeology shows that exponentiation is first introduced as a representation of multiplicative growth, subsequently generalized through pattern-based reasoning, and finally institutionalized as a system of mathematical tools. This progression reflects broader perspectives on the construction of mathematical knowledge, which emphasize the transformation of situated meanings into increasingly formal and abstract structures (Ernest, 2003). The resulting learning trajectory is consistent with descriptions of mathematical development as a gradual reorganization of knowledge across interconnected stages of learning (Weber et al., 2015). Consequently, knowledge of exponents emerges not as a collection of isolated rules but as a coherent mathematical organization linking meaning, generalization, justification, and application.

CONCLUSIONS

This study reconstructed the praxeological organization of exponent learning in a Grade 10 mathematics textbook through the lens of the Anthropological Theory of the Didactic. The analysis revealed that knowledge of exponents is organized into a coherent progression that moves from interpreting multiplicative growth situations to formulating exponent rules and, ultimately, to institutionalizing exponent properties as tools for mathematical activity. Rather than presenting exponentiation as a collection of isolated procedures, the textbook establishes connections between contextual reasoning, algebraic generalization, and formal symbolic practice. At the global level, the reconstructed organization shows that the properties of exponents gradually evolve from objects of mathematical construction into instruments for solving equations and simplifying expressions. This progression contributes to the structured development of knowledge of exponents by linking meaning construction, generalization, and procedural application within a single mathematical framework. However, the analysis also indicates that the visibility of technological explanations decreases as students move toward more formal tasks, resulting in an increasing prominence of procedural techniques during the institutionalization phase. From an ATD perspective, these findings demonstrate how textbook tasks shape the balance between praxis and logos in the development of mathematical knowledge. The study contributes to the growing body of textbook research by showing how exponent learning is organized not only through the selection of mathematical content but also through the sequencing of praxeological components that govern students' opportunities to construct, justify, and apply mathematical ideas. Future research may extend this approach to other mathematical domains and examine how different textbook organizations influence the development of conceptual and procedural understanding.

ADDITIONAL INFORMATION

Section	Description
Funding	This research received no external funding.
Acknowledgment	The authors would like to express their sincere gratitude to the Mathematics Education Study Program, Faculty of Teacher Training and Education, Universitas Singaperbangsa Karawang, for its support of this research. The authors also thank all parties who contributed to the completion of this study.
Conflict of Interest	The authors declare no conflict of interest.
Data Availability	The data supporting the findings of this study are available upon reasonable request from the corresponding author.
Author Contributions	Conceptualization, L.C., S.Y.M., and M.P.; methodology, L.C., S.Y.M., and M.P.; data collection and analysis, L.C.; writing—original draft preparation, L.C.; writing—review and editing, S.Y.M. and M.P.; supervision, S.Y.M. and M.P. All authors have read and agreed to the published version of the manuscript.

REFERENCES

Abou-Hayt, I. (2026). A critical look at the anthropological theory of the didactic. *Research in Mathematics Education, 28*(1), 3–24. https://doi.org/10.1080/14794802.2024.2344754

Amit, M., & Neria, D. (2008). Rising to the challenge: Using generalization in pattern problems to unearth the algebraic skills of talented pre-algebra students. *ZDM Mathematics Education, 40*(1), 111–129. https://doi.org/10.1007/s11858-007-0069-5

Bani Irshid, M. M., Khasawneh, A. A., & Al-Barakat, A. A. (2023). The effect of conceptual understanding principles-based training program on enhancement of pedagogical knowledge of mathematics teachers. *Eurasia Journal of Mathematics, Science and Technology Education, 19*(6), Article em2277. https://doi.org/10.29333/ejmste/13215

Baroody, A. J. (2013). The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills* (pp. 1–33). Routledge. https://doi.org/10.4324/9781410607218-1

Bittar, M. (2021). Overview of research on textbooks in Brazilian compulsory education. In B. Barquero, I. Florensa, P. Nicolás, & N. Ruiz-Munzón (Eds.), *Extended abstracts spring 2019* (Vol. 13). Birkhäuser. https://doi.org/10.1007/978-3-030-76413-5_15

Borji, V., Radmehr, F., & Font, V. (2021). The impact of procedural and conceptual teaching on students' mathematical performance over time. *International Journal of Mathematical Education in Science and Technology, 52*(3), 404–426. https://doi.org/10.1080/0020739X.2019.1688404

Bosch, M. (2015). Doing research within the anthropological theory of the didactic: The case of school algebra. In S. Cho (Ed.), *Selected regular lectures from the 12th International Congress on Mathematical Education* (pp. 51–71). Springer. https://doi.org/10.1007/978-3-319-17187-6_4

Bosch, M., Chevallard, Y., García, F. J., & Monaghan, J. (2020). *Working with the anthropological theory of the didactic in mathematics education*. Routledge. https://doi.org/10.4324/9780429198168

Bowen, G. A. (2009). Document analysis as a qualitative research method. *Qualitative Research Journal, 9*(2), 27–40. https://doi.org/10.3316/QRJ0902027

Carraher, D. W., Martinez, M. V., & Schliemann, A. D. (2008). Early algebra and mathematical generalization. *ZDM Mathematics Education, 40*(1), 3–22. https://doi.org/10.1007/s11858-007-0067-7

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic an attempt at a principled approach. *Hiroshima journal of mathematics education*, *12*, 71-114. https://doi.org/10.24529/hjme.1205

Chevallard, Y. (2022). Challenges and advances in teacher education within the anthropological theory of the didactic. In Y. Chevallard et al. (Eds.), *Advances in the anthropological theory of the didactic*. Birkhäuser. https://doi.org/10.1007/978-3-030-76791-4_7

Chevallard, Y., Bosch, M. (2020). Didactic Transposition in Mathematics Education. In: Lerman, S. (eds) Encyclopedia of Mathematics Education. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_48

Ciccione, L., Sablé-Meyer, M., & Dehaene, S. (2022). Analyzing the misperception of exponential growth in graphs. *Cognition, 225*, Article 105112. https://doi.org/10.1016/j.cognition.2022.105112

Confrey, J. (1991). The concept of exponential functions: A student's perspective. In L. P. Steffe (Ed.), *Epistemological foundations of mathematical experience* (pp. 124–159). Springer. https://doi.org/10.1007/978-1-4612-3178-3_8

Confrey, J., & Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. *Educational Studies in Mathematics, 26*(2–3), 135–164. https://doi.org/10.1007/BF01273661

Ernest, P. (2003). *Constructing mathematical knowledge: Epistemology and mathematics education*. Routledge. https://doi.org/10.4324/9780203454206

Gascón, J. (2024). Contributions of the anthropological theory of the didactic to the epistemological programme of research in mathematics education. *ZDM–Mathematics Education, 56*, 1319–1330. https://doi.org/10.1007/s11858-024-01563-1

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. *Handbook of qualitative research*, *2*(163-194), 105. https://miguelangelmartinez.net/IMG/pdf/1994_Guba_Lincoln_Paradigms_Quali_Research_chapter.pdf

Hackenberg, A. J. (2010). Students' reasoning with reversible multiplicative relationships. *Cognition and Instruction, 28*(4), 383–432. https://doi.org/10.1080/07370008.2010.511565

Ismayanti, S., Jatisunda, M. G., & Hoon, T. S. (2025). Didactic transposition of trigonometric ratios: A comparative study of school and university textbooks. *Journal on Mathematics Education, 16*(4), 1283–1312. https://doi.org/10.22342/jme.v16i4.pp1283-1312

Jatisunda, M. G., Hoon, T. S., Jamilah, J., & Rohimatunissa, D. (2025). Praxeological analysis of junior secondary students' epistemological obstacles in algebraic operations. *Journal of Research and Advances in Mathematics Education, 10*(4), 218–230. https://doi.org/10.23917/jramathedu.v10i4.13374

Keazer, L., & Phaiah, J. (2023). Analyzing prospective elementary teachers' evidence of conceptual understanding and procedural fluency. *Investigations in Mathematics Learning, 15*(2), 135–148. https://doi.org/10.1080/19477503.2022.2139112

Kieran, C. (2013). The false dichotomy in mathematics education between conceptual understanding and procedural skills: An example from algebra. In K. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 153–171). Springer. https://doi.org/10.1007/978-1-4614-6977-3_7

Kowiyah, K., Mulyawati, I., & Umam, K. (2019). Conceptual understanding and mathematical representation analysis of realistic mathematics education based on personality types. *Al-Jabar: Jurnal Pendidikan Matematika, 10*(2), 201–210. https://doi.org/10.24042/ajpm.v10i2.4605

Pansell, A. (2023). Mathematical knowledge for teaching as a didactic praxeology. *Frontiers in Education, 8*, Article 1165977. https://doi.org/10.3389/feduc.2023.1165977

Pepin, B., Gueudet, G. (2020). Curriculum Resources and Textbooks in Mathematics Education. In: Lerman, S. (eds) Encyclopedia of Mathematics Education. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_40

Peters, J. (2022). Modifying exercises in mathematics service courses for engineers based on subject-specific analyses of engineering mathematical practices. In R. Biehler, M. Liebendörfer, G. Gueudet, C. Rasmussen, & C. Winsløw (Eds.), *Practice-oriented research in tertiary mathematics education* (Advances in Mathematics Education). Springer. https://doi.org/10.1007/978-3-031-14175-1_28

Polly, D., & Martin, C. S. (2025). Build procedural fluency from conceptual understanding. https://doi.org/10.1108/978-1-83708-512-520251042

Pradipta, T. R., Turmudi, T., Hendriyanto, A., & Pramartha, I. N. B. (2025). Praxeological analysis of number patterns through the anthropological theory of the didactic perspective. *International Journal of Science and Mathematics Education, 8*(3), 799–819. https://doi.org/10.24042/0rqrd137

Rivera, F. D. (2018). Pattern generalization processing of elementary students: Cognitive factors affecting the development of exact mathematical structures. *Eurasia Journal of Mathematics,* *Science and Technology Education, 14*(9), Article em1586. https://doi.org/10.29333/ejmste/92554

Şenay, Ş. C. (2024). Analysis of misconceptions and errors regarding exponential and radical expressions through the theory of reducing abstraction. *Research on Education and Psychology, 8*(2), 281–295. https://doi.org/10.54535/rep.1520588

Suarka, N. K. S. D., & Kusumah, Y. S. (2024). Students' learning obstacles in exponential: A case study in Indonesian higher education students. *Jurnal Pendidikan MIPA, 25*(2), 530–541. https://doi.org/10.23960/jpmipa/v25i2.pp530-541

Sunzuma, G. (2026). Cultural Activities in Secondary School Mathematics Textbooks in Zimbabwe: Toward a Contextualized Curriculum. In: Ampadu, E., Jojo, Z., Hartell, E., Sunzuma, G., Mensah, F.S. (eds) Mathematics for All. Springer, Cham. https://doi.org/10.1007/978-3-032-22072-1_6

Susanto, D., Kurniawan, T., Sihombing, S. K., Salim, E., Magdalena, M., Radjawane, R., Salmah, U., & Wardani, A. K. (2021). *Matematika*. Pusat Kurikulum dan Perbukuan, Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi. https://buku.kemdikbud.go.id

Syafiqoh, N., Amin, S. M., & Siswono, T. Y. E. (2018). Analysis of student's understanding of exponential concept: A perspective of Pirie-Kieren theory. *Journal of Physics: Conference Series, 1108*(1), Article 012022. https://doi.org/10.1088/1742-6596/1108/1/012022

Ulusoy, F. (2019). Serious obstacles hindering middle school students' understanding of integer exponents. *International Journal of Research in Education and Science, 5*(1), 52–69. https://files.eric.ed.gov/fulltext/EJ1198050.pdf

Vinner, S. (2020). Concept development in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Springer. https://doi.org/10.1007/978-3-030-15789-0_29

Weber, E., Walkington, C., & McGalliard, W. (2015). Expanding notions of learning trajectories in mathematics education. *Mathematical Thinking and Learning, 17*(4), 253–272. https://doi.org/10.1080/10986065.2015.1083836

Wijayanti, D., & Aufa, D. N. (2020). Picturing textbook on exponent equations based on praxeology organization. In *Proceedings of the 2nd Social and Humaniora Research Symposium (SoRes 2019)* (pp. 494–498). Atlantis Press. https://doi.org/10.2991/assehr.k.200225.107

Yunianta, T. N. H., Suryadi, D., Dasari, D., & Herman, T. (2023). Textbook praxeological-didactical analysis: Lessons learned from the Indonesian mathematics textbook. *Journal on Mathematics Education, 14*(3), 503–524. https://doi.org/10.22342/jme.v14i3.pp503-524

Zamanabadi, S., & Rafiepour, A. (2026). Application of anthropological theory of didactic in teaching of mathematics. *Mathematics and Society, 11*(1), 25–41. https://doi.org/10.22108/msci.2025.143082.1718

Zeynivandnezhad, F., Saralar-Aras, I., & Halai, A. (2024). A refined framework for qualitative content analysis of mathematics textbooks. *Eurasia Journal of Mathematics, Science and Technology Education, 20*(3), em2412. https://doi.org/10.29333/ejmste/14284