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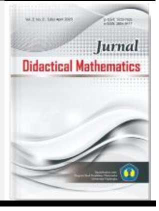

Examining Students' Conceptual Understanding in Solving Contextual Problems on Linear Equations in One Variable: A Cross-Case Qualitative Analysis

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Abstract

Mathematical conceptual understanding plays a crucial role in enabling students to establish meaningful relationships among mathematical ideas and apply knowledge flexibly in problem-solving situations. However, many students continue to rely on procedural approaches without developing deeper conceptual reasoning, particularly when solving contextual mathematical problems. Existing studies have mainly focused on achievement outcomes or isolated dimensions of conceptual understanding, providing limited insight into how students demonstrate conceptual understanding across multiple indicators within contextual problem-solving situations. Therefore, this study aimed to analyze students' conceptual understanding in solving contextual problems related to linear equations in one variable across different levels of ability. This study employed a descriptive qualitative research design involving 30 eighth-grade students at SMP Negeri 1 Jelimpo, West Kalimantan, Indonesia. Data were collected through written tests and semi-structured interviews. Participants representing high-, medium-, and low-level conceptual understanding were selected using purposive sampling techniques. Data were analyzed through data reduction, data display, and conclusion drawing using triangulation procedures. The findings revealed that students' conceptual understanding demonstrated progressive differences in cognitive organization rather than merely differences in procedural performance. High-level students exhibited integrated conceptual structures, medium-level students demonstrated transitional characteristics, and low-level students showed fragmented understanding characterized by symbolic dependence and procedural imitation. Furthermore, conceptual understanding indicators functioned as interconnected dimensions in which difficulties in one indicator influenced performance in other dimensions. The findings highlight the importance of instructional practices emphasizing conceptual exploration, multiple representations, contextual learning, and structured scaffolding.

INTRODUCTION

Mathematical learning extends beyond the acquisition of computational procedures because it fundamentally involves the construction of meaning through interconnected conceptual structures. Within mathematics education, conceptual understanding has consistently been recognized as an essential component of mathematical proficiency because it enables learners to establish relationships among ideas and transfer knowledge across different situations. According to Vinner (2020), concept development in mathematics does not emerge through isolated memorization of formulas or definitions; rather, it develops progressively through interactions between learners' prior cognitive structures and newly encountered mathematical experiences. This perspective suggests that understanding evolves as students continuously reorganize and refine their conceptual schemas. Similarly, Malatjie and Machaba (2019) explained that students who successfully establish conceptual relationships demonstrate stronger abilities to interpret mathematical ideas and communicate them meaningfully. Therefore, conceptual understanding should be viewed not merely as an instructional outcome but as a cognitive process through which learners construct coherent knowledge systems that support flexible mathematical thinking.

Although conceptual understanding is considered central to mathematics learning, instructional practices in many educational settings continue to emphasize procedural proficiency and algorithmic mastery. This instructional tendency frequently positions mathematics as a set of rules to be memorized rather than as a system of meaningful relationships to be understood. Star (2020) distinguished instrumental understanding, which emphasizes procedural application, from relational understanding, which involves understanding why mathematical relationships operate in specific ways. While procedural fluency contributes to efficiency in solving routine tasks, Braithwaite and Sprague (2021) argued that meaningful mathematical performance requires the interaction of conceptual knowledge, procedural knowledge, and metacognitive processes. Without such integration, students often become dependent on memorized procedures and demonstrate limited adaptability in unfamiliar situations. Supporting this argument, Al-Mutawah et al. (2019) found that students possessing stronger conceptual understanding demonstrated superior problem-solving performance because they interpreted mathematical structures conceptually rather than relying exclusively on algorithmic application.

The limitations of procedural dependence become increasingly visible in algebra learning, particularly in linear equations in one variable. As one of the fundamental topics in secondary mathematics, linear equations require students not only to manipulate symbols but also to understand relationships among variables, coefficients, constants, and symbolic representations. However, previous studies have shown that students frequently experience conceptual difficulties in algebraic learning situations. For example, Yansa et al. (2021) reported that misconceptions regarding variables and algebraic representations often lead students to interpret mathematical symbols superficially rather than understand their conceptual meanings. Similarly, Nurmawanti et al. (2021) found that students frequently relied on imitative reasoning in which procedures were reproduced without meaningful conceptual interpretation. Additionally, Fatio et al. (2020) and Ernawati and Muzaini (2020) demonstrated that students often struggle to organize mathematical information systematically and transform contextual situations into appropriate mathematical representations.

Understanding such conceptual difficulties requires attention not only to students' final answers but also to the characteristics of their conceptual thinking processes. Conceptual understanding in mathematics is commonly reflected through several observable indicators that reveal how knowledge has been organized and internalized cognitively. Suharto and Widada (2019) explained that students' cognitive structures influence the manner in which mathematical concepts are interpreted and connected within existing knowledge systems. Therefore, conceptual understanding can be identified through students' abilities to restate concepts, classify mathematical objects according to their properties, provide examples and non-examples, and represent concepts through multiple mathematical forms. Furthermore, Kharis et al. (2021) suggested that these dimensions contribute directly to students' mathematical reasoning and literacy because they facilitate meaningful interpretation and communication of mathematical ideas.

The development of conceptual understanding may become more apparent within contextual learning environments because contextual problems encourage students to establish connections between mathematical concepts and meaningful experiences. Rather than functioning solely as motivational tools, contextual situations may serve as conceptual foundations that support knowledge construction. Reinke (2020) described contextual problems as conceptual anchors that help students attach mathematical meaning to real-world experiences. Similarly, Kohen and Nitzan-Tamar (2022) emphasized that contextual mathematical modelling requires students to integrate interpretation, representation, and reasoning processes in constructing mathematical solutions. Supporting this perspective, previous empirical studies indicated that contextual approaches contribute positively to students' mathematical reasoning and problem-solving abilities by encouraging learners to connect prior knowledge with new experiences (Basid et al., 2024; Hadi et al., 2024; Rohimatunisa & Sudianto, 2023).

Although numerous studies have investigated students' conceptual understanding in mathematics, existing research has predominantly focused on achievement outcomes or isolated dimensions of understanding. Previous investigations have frequently provided quantitative descriptions of learning outcomes while offering limited qualitative explanations regarding how students demonstrate conceptual understanding across multiple indicators during contextual problem-solving processes. Consequently, limited evidence remains regarding students' conceptual characteristics across different levels of understanding within contextual mathematical tasks. Addressing this issue is important because understanding students' conceptual profiles may provide richer insights into students' reasoning processes and support the development of instructional strategies that emphasize meaningful learning experiences. Therefore, this study aims to analyze students' conceptual understanding in solving contextual problems related to linear equations in one variable by examining multiple indicators of conceptual understanding across different levels of student ability.

METHODS

Research Design

This study employed a descriptive qualitative research design to investigate students' conceptual understanding in solving contextual problems related to linear equations in one variable. A qualitative approach was considered appropriate because the purpose of this study was not to examine causal relationships or test hypotheses but to obtain an in-depth understanding of students' conceptual thinking and reasoning processes during mathematical problem-solving activities. Qualitative research is particularly suitable when the focus of investigation concerns participants' experiences, interpretations, and meaning-making processes within authentic educational contexts. In mathematics education research, qualitative approaches have increasingly been utilized to investigate how learners construct mathematical knowledge and conceptual understanding because such approaches allow researchers to capture the complexity of students' cognitive processes beyond numerical measurements (Gqoli et al., 2023). Furthermore, Karamık and Bağ (2024) emphasized that qualitative methods in mathematics education facilitate a more comprehensive understanding of participants' perspectives and reasoning processes. Similarly, Akcam et al. (2019) argued that qualitative designs provide opportunities to explore dynamic relationships and interpret complex phenomena that may not be adequately explained through quantitative approaches. Therefore, a descriptive qualitative design was considered appropriate for examining the characteristics of students' conceptual understanding within contextual mathematical situations.

Research Setting and Participants

The study was conducted at SMP Negeri 1 Jelimpo, located in Landak Regency, West Kalimantan, Indonesia. Participants consisted of 30 eighth-grade students from class VIII D who had completed learning activities related to linear equations in one variable as part of the mathematics curriculum. The class was selected because students had received formal instruction on the targeted topic and represented diverse academic characteristics and learning abilities.

Participant Selection Procedure

Participants involved in the in-depth analysis were selected using purposive sampling techniques. In qualitative research, participant selection aims to maximize information richness rather than statistical representativeness. According to Tajik et al. (2025), purposive sampling enables researchers to select participants possessing characteristics relevant to the objectives of the study. Likewise, Ames et al. (2019) explained that purposive sampling facilitates the identification of information-rich cases capable of providing deeper insights into the investigated phenomenon. Based on these considerations, participant selection was conducted using three criteria: (1) students belonged to class VIII D, (2) students had studied linear equations in one variable, and (3) students represented high-, medium-, and low-level conceptual understanding based on initial written assessment results. The selected participants were subsequently used for a detailed analysis of conceptual understanding characteristics across different ability levels.

Research Instruments

Written Test

Data were collected using written tests and semi-structured interviews to obtain comprehensive information regarding students' conceptual understanding. The written test consisted of four contextual essay questions designed to assess students' conceptual understanding of linear equations in one variable. The development of the written instrument was guided by indicators of conceptual understanding, including: (1) restating concepts, (2) classifying mathematical objects according to their properties, (3) generating examples and non-examples, and (4) representing concepts through multiple mathematical forms. These indicators were selected because conceptual learning in mathematics involves not only acquiring factual knowledge but also constructing relationships among mathematical ideas and representations. Previous studies have indicated that conceptual understanding contributes significantly to students' abilities to establish mathematical connections and solve problems meaningfully (Kenedi et al., 2019; Ulfa & Puspaningtyas, 2020). Furthermore, Mu et al. (2022) emphasized that mathematics assessment should capture students' conceptual performance through multiple dimensions of understanding rather than relying solely on procedural outcomes. Prior to implementation, the instrument underwent expert validation procedures to ensure conceptual appropriateness and content relevance. Content validity was established following principles outlined by Ulya et al. (2024), emphasizing consistency between assessment indicators and intended learning objectives.

Semi-Structured Interview

Semi-structured interviews were subsequently conducted to obtain more detailed insights into students' reasoning processes and conceptual characteristics underlying their written responses. The interview protocol was developed according to the same conceptual understanding indicators employed in the written assessment to ensure consistency across data sources. Semi-structured interviews were selected because they provide a structured framework while simultaneously allowing participants to explain experiences and reasoning processes in greater depth. According to Brown and Danaher (2019), semi-structured interviews facilitate authentic and dialogical interactions that enable participants to express perspectives more freely while maintaining the focus of inquiry. Individual interviews were conducted with selected participants and lasted approximately 15–25 minutes. All interview sessions were audio-recorded and transcribed verbatim to facilitate detailed examination of students' conceptual explanations and reasoning processes.

Trustworthiness and Data Validation

To ensure the trustworthiness and credibility of the findings, methodological rigor was established through triangulation techniques and systematic validation procedures. Trustworthiness in qualitative research involves ensuring that findings accurately represent participants' experiences and interpretations rather than reflecting the researcher's assumptions. Ahmed (2024) emphasized that credibility, dependability, confirmability, and transferability constitute essential pillars of trustworthiness in qualitative inquiry. Therefore, methodological triangulation was conducted by comparing findings obtained from written tests and interviews, while source triangulation involved examining information collected from multiple participants and supporting informants. These procedures were intended to strengthen the credibility and consistency of the findings.

Data Analysis Procedure

Data analysis followed the interactive analytical framework consisting of data reduction, data display, and conclusion drawing and verification. During data reduction, relevant information from written responses and interview transcripts was selected, organized, and coded according to conceptual understanding indicators. According to Kalpokaite and Radivojevic (2019), systematic qualitative analysis allows researchers to identify meaningful patterns within complex datasets. Likewise, Salmona and Kaczynski (2024) emphasized that qualitative analytical strategies should facilitate the interpretation of relationships and emerging conceptual themes. Subsequently, the data were displayed in descriptive and interpretative forms to identify recurring patterns and relationships among conceptual understanding indicators. Finally, conclusions were continuously drawn and verified through iterative interpretation processes to ensure that findings remained credible, coherent, and grounded in the collected evidence.

RESULT

This section presents findings regarding students' conceptual understanding in solving contextual problems related to linear equations in one variable. Data were obtained from written assessments administered to 30 students and semi-structured interviews conducted with selected participants representing high, medium, and low levels of conceptual understanding. Analysis was conducted based on four conceptual understanding indicators: (1) restating concepts, (2) classifying mathematical objects according to their properties, (3) providing examples and non-examples, and (4) representing concepts using multiple mathematical forms. The findings presented below integrate written responses and interview data to provide a comprehensive description of students' conceptual characteristics.

Distribution of Students' Conceptual Understanding

Students' levels of conceptual understanding were initially identified through written test results administered to all participants. The assessment scores were analyzed to determine variations in students' conceptual performance and to classify students according to their levels of understanding. This categorization process was conducted to provide an overview of the distribution of conceptual understanding among participants and to support the selection of information-rich participants for further in-depth analysis through interviews. Based on the obtained scores, students were grouped into high-, medium-, and low-level categories according to predetermined criteria. The distribution of students across these categories is presented in Table 1.

Table 1. *Distribution of Students' Conceptual Understanding Levels*

Category	Score Range	Number of Students	Percentage
High	75–100	7	23.3
Medium	60–74	15	50.0
Low	0–59	8	26.7
Total		30	100

The findings indicate that the majority of students were classified within the medium category (50%), while approximately one-fourth of students demonstrated low conceptual understanding. Students representing each category were subsequently selected for further analysis through purposive sampling: AJ (high category), DD (medium category), and WW (low category).

Performance Across Conceptual Understanding Indicators

Following the categorization of students' conceptual understanding levels, further analysis was conducted to examine students' performance across each conceptual understanding indicator. The analysis aimed to identify the extent to which students demonstrated specific conceptual characteristics during the problem-solving process. The four indicators analyzed in this study included the ability to restate concepts, classify mathematical objects according to their properties, provide examples and non-examples, and represent concepts using multiple mathematical forms. Examining these indicators allowed a more detailed understanding of students' conceptual performance beyond overall achievement scores and provided insights into specific strengths and difficulties demonstrated by students across different levels of understanding. The results of students' performance across the conceptual understanding indicators are presented in Table 2.

Table 2. *Students' Performance Across Conceptual Understanding Indicators*

Indicators	High	Medium	Low
Restating concepts	✓	✓	✗
Classifying mathematical objects	✓	Partial	✗
Providing examples and non-examples	✓	✗	✗
Multiple mathematical representations	✓	Partial	✗

The findings reveal that high-level students fulfilled all conceptual understanding indicators. Medium-level students demonstrated partial performance in several indicators, particularly in mathematical representation and classification tasks. In contrast, low-level students demonstrated substantial difficulties across all indicators.

Conceptual Understanding Profile: High Category Student

Students categorized within the high-level group demonstrated comprehensive conceptual understanding across all indicators examined in this study. Analysis of written responses revealed that the student accurately identified relevant mathematical information, distinguished mathematical elements appropriately, generated additional examples and non-examples, and represented contextual information using algebraic expressions. The student's response reflected not only procedural correctness but also a coherent understanding of relationships among mathematical concepts. Findings from the written assessment and interview data were subsequently compared through triangulation procedures to examine the consistency of conceptual indicators.

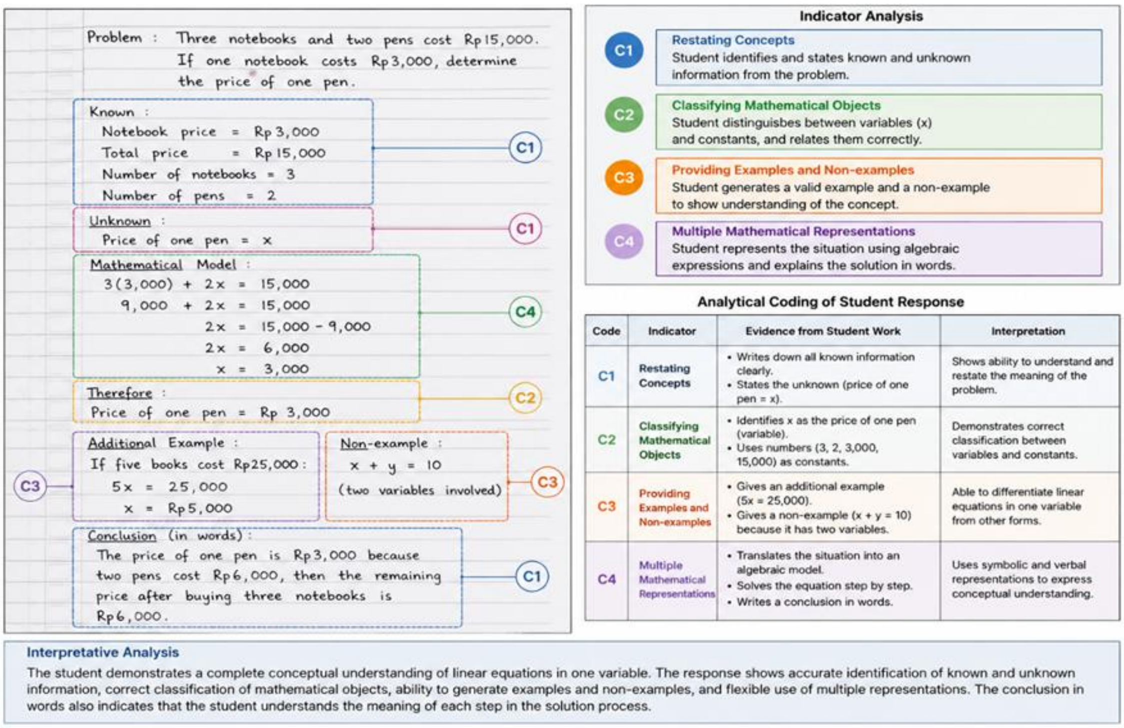

Figure 1. *Written Response of High-Level Student (AJ): Conceptual Understanding Across Multiple Indicators*

The written response demonstrates that the student systematically organized information from the contextual problem before constructing a mathematical model. The student first identified known and unknown information and represented the unknown quantity using the variable x. Furthermore, the student correctly translated the contextual situation into the algebraic equation $3(3{,}000) + 2x = 15{,}000$, indicating an understanding of relationships among variables, coefficients, and constants. Unlike procedural responses that merely perform calculations, the student also generated an additional example and identified a non-example by distinguishing equations involving one variable from expressions containing multiple variables.

To obtain a more systematic interpretation of the student's conceptual performance, the written response was further analyzed using the coding framework developed from the conceptual understanding indicators employed in this study. The coding process aimed to identify how specific evidence from the student's response reflected different dimensions of conceptual understanding and to determine the extent to which each indicator was demonstrated during the problem-solving process. Through this analytical procedure, the observed written evidence was organized and linked to corresponding conceptual indicators, allowing a more detailed examination of the student's conceptual characteristics. The coding results for the high-level student are presented in Table 3.

Table 3. *Analytical Coding of High-Level Student Responses*

Code	Evidence from Student Response	Conceptual Understanding Indicator
C1	Identified known and unknown information correctly	Restating concepts
C2	Distinguished variables, coefficients, and constants appropriately	Classifying mathematical objects
C3	Generated an additional example and identified a non-example	Providing examples and non-examples
C4	Constructed algebraic equations and interpreted results verbally	Multiple mathematical representations

The coding results indicate that all conceptual understanding indicators were fulfilled by the high-level student. Evidence for the first indicator was observed through the student's ability to restate the problem situation by identifying relevant information explicitly. The second indicator emerged when the student differentiated mathematical elements according to their functions within the equation. The third indicator was reflected in the student's ability to construct additional examples and distinguish non-examples based on conceptual characteristics. Finally, the fourth indicator appeared in the student's ability to transform contextual information into symbolic representations and explain the meaning of the obtained solution. Interview findings further supported these observations. During the interview session, the student explained:

"I first identified what was unknown and represented it as x because x can represent the price of one item. Then I looked at the total price and connected it to the number of items."

The interview excerpt suggests that the student did not merely apply memorized procedures but demonstrated an understanding of the conceptual meaning underlying symbolic representations. The explanation also indicates that the student understood the relationship between contextual information and mathematical structures. Therefore, triangulation between written responses and interview findings confirms that the student demonstrated integrated conceptual understanding characterized by meaningful interpretation, representational flexibility, and coherent reasoning processes.

Conceptual Understanding Profile: Medium Category Student

Students categorized within the medium-level group demonstrated partial conceptual understanding across the indicators examined in this study. Analysis of written responses indicated that students were generally able to identify relevant information and construct basic mathematical models; however, their conceptual understanding remained incomplete and less flexible than that demonstrated by students in the high category. Students could identify known and unknown information from contextual situations but experienced difficulties when extending their understanding beyond routine procedural applications. Furthermore, several responses revealed inconsistencies in organizing mathematical information and explaining conceptual relationships underlying the solution process. Triangulation between written responses and interview findings was subsequently conducted to examine the consistency of students' conceptual performance.

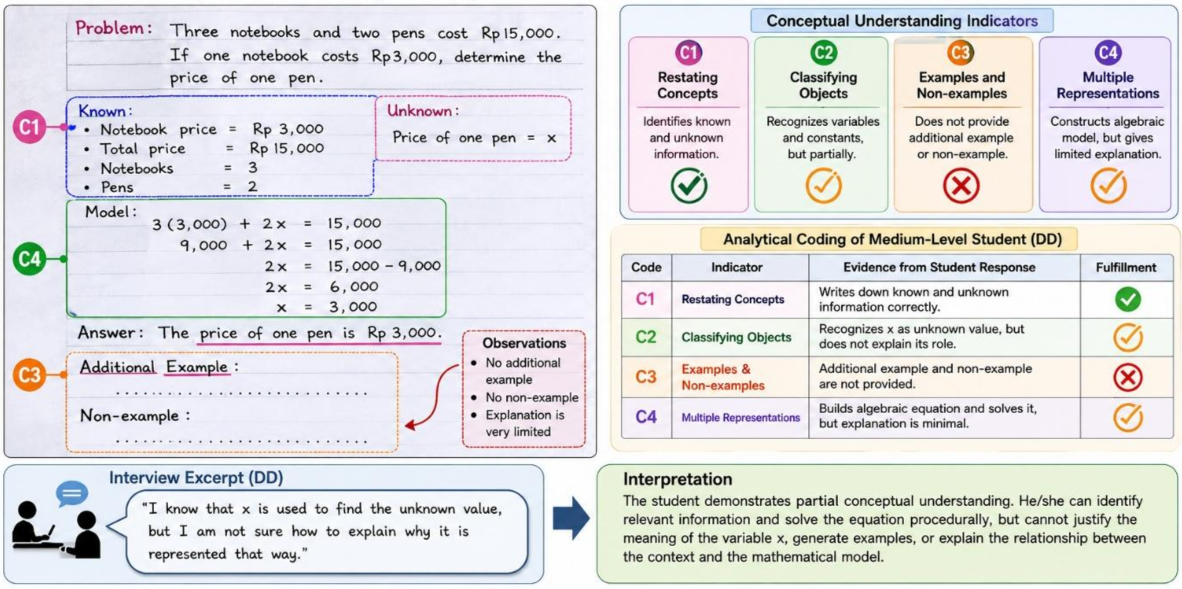

Figure 2. *Written Response of Medium-Level Student (DD): Partial Conceptual Understanding in Solving Contextual Linear Equation Problems*

The written response indicates that the student identified essential information from the contextual problem and represented the unknown quantity using a variable. The student was also able to construct an algebraic equation and perform procedural calculations correctly. However, the response lacked conceptual explanations regarding why the mathematical model was formed in a particular way. In addition, the student did not generate additional examples or distinguish examples from non-examples. The written work also demonstrated limited representational flexibility because the solution process focused mainly on symbolic procedures without further interpretation.

To provide a more structured interpretation of the student's conceptual performance, the written response was coded according to the conceptual understanding indicators employed in this study. The coding results for the medium-level student are presented in Table 5.

Table 5. *Analytical Coding of Medium-Level Student Responses*

Code	Evidence from Student Response	Conceptual Understanding Indicator
C1	Identified known and unknown information appropriately	Restating concepts
C2	Recognized variables and mathematical elements, although partially	Classifying mathematical objects
C3	Did not provide additional examples or non-examples	Providing examples and non-examples
C4	Constructed algebraic representations but with limited explanation	Multiple mathematical representations

The coding results indicate that the student partially fulfilled three conceptual understanding indicators. Evidence of the first indicator was observed through the student's ability to restate information from the problem context. The second indicator emerged because the student identified mathematical elements such as variables and constants, although relationships among those elements were not fully explained. Difficulties became more apparent in the third indicator, where the student failed to generate examples and non-examples that reflected conceptual understanding. Similarly, the fourth indicator was only partially achieved because the student translated contextual information into symbolic forms but did not provide sufficient explanation of the meaning represented by the solution. Interview findings further supported these observations. During the interview session, the student stated:

> *"I know that x is used to find the unknown value, but I am not sure how to explain why it is represented that way."*

This statement indicates that the student relied primarily on procedural familiarity rather than conceptual interpretation. Although the student understood that variables were associated with unknown values, the inability to explain the underlying reasoning suggests that symbolic representations had not yet been fully integrated into a coherent conceptual structure. Therefore, triangulation between written responses and interview findings indicates that students within the medium category demonstrated transitional conceptual understanding characterized by procedural competence accompanied by limited conceptual depth and representational flexibility.

Conceptual Understanding Profile: Low Category Student

Students categorized within the low-level group demonstrated substantial conceptual difficulties across all conceptual understanding indicators examined in this study. Analysis of written responses indicated that students experienced difficulties in identifying relevant mathematical information and frequently relied on direct symbolic manipulation without understanding the conceptual meaning underlying mathematical expressions. Unlike students in the high and medium categories, students in this category showed limited ability to organize contextual information systematically and frequently applied mathematical symbols based on memorized patterns rather than conceptual reasoning. To ensure consistency between data sources, findings from written assessments were compared with interview data through triangulation procedures.

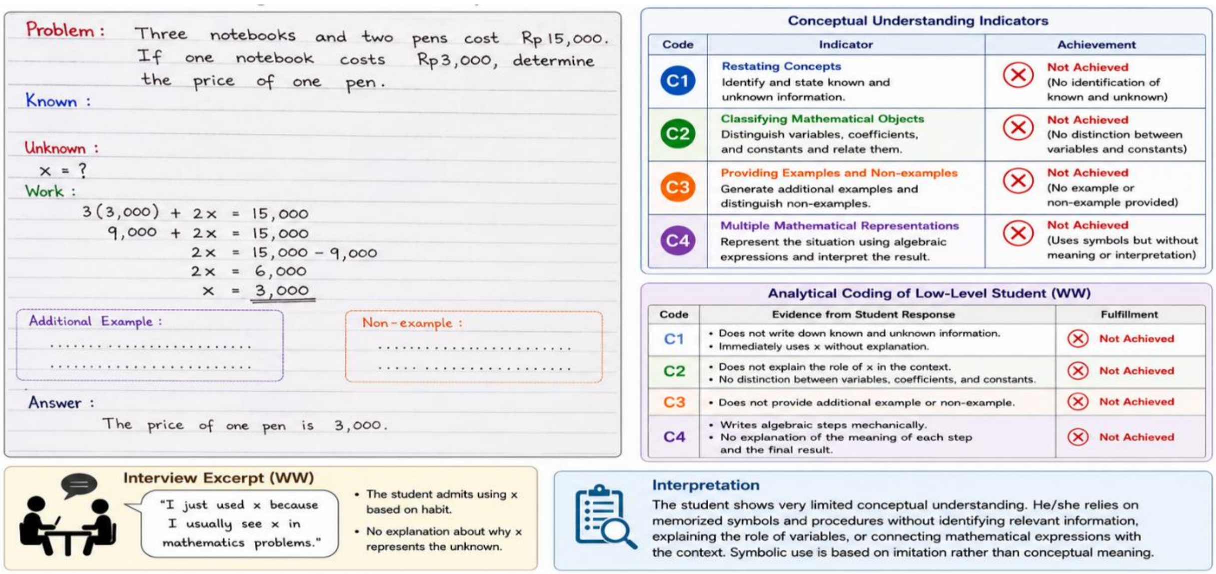

Figure 3. *Written Response of Low-Level Student (WW): Symbolic Dependence and Limited Conceptual Understanding in Solving Contextual Linear Equation Problems*

The written response indicates that the student directly used algebraic symbols without first identifying known and unknown information from the contextual problem. The student immediately represented the situation using the variable x without explaining the relationship between the variable and the contextual information provided. In addition, procedural steps were incomplete and lacked meaningful explanation. The response also did not include additional examples, non-examples, or verbal interpretation of the obtained result. Such findings suggest that the student focused primarily on surface-level procedures rather than conceptual understanding.

To provide a clearer interpretation of the student's conceptual characteristics, the written response was coded according to the conceptual understanding indicators employed in this study. The coding results for the low-level student are presented in Table 6.

Table 6. *Analytical Coding of Low-Level Student Responses*

Code	Evidence from Student Response	Conceptual Understanding Indicator
C1	Unable to identify known and unknown information systematically	Restating concepts
C2	Did not distinguish variables, coefficients, and constants appropriately	Classifying mathematical objects
C3	Failed to generate examples and non-examples	Providing examples and non-examples
C4	Used symbolic forms without explanation or contextual interpretation	Multiple mathematical representations

The coding results indicate that none of the conceptual understanding indicators were fully achieved by the student. Difficulties in the first indicator were reflected in the inability to restate relevant information from the contextual problem. Similarly, the second indicator was not fulfilled because mathematical elements were used without understanding their functions and relationships. The third indicator was absent because the student did not provide additional examples or distinguish valid and invalid cases. Furthermore, the fourth indicator was not achieved because symbolic representations were written mechanically without meaningful explanation of the mathematical relationships involved.

Interview findings further supported these observations. During the interview session, the student stated:

"I just used x because I usually see x in mathematics problems."

The interview excerpt indicates that the student's use of variables was based primarily on memorization and habitual exposure rather than conceptual interpretation. The student appeared to associate mathematical symbols with routine procedures without understanding the representational meaning of those symbols within contextual situations. Therefore, triangulation between written responses and interview findings suggests that students in the low category demonstrated fragmented conceptual understanding characterized by symbolic dependence, procedural imitation, and limited representational awareness.

Cross-Case Analysis of Conceptual Understanding

To obtain a broader understanding of students' conceptual characteristics, findings from written responses and interview data were synthesized through cross-case analysis procedures. This analysis aimed to identify patterns of conceptual understanding across high-, medium-, and low-level groups by examining similarities and differences in students' conceptual performance. The comparison was conducted based on the four conceptual understanding indicators employed in this study: restating concepts, classifying mathematical objects, providing examples and non-examples, and representing concepts through multiple mathematical forms. Through this comparison, the analysis provides a comprehensive view of how conceptual understanding varied across different levels of ability. The results of the cross-case analysis are presented in Table 7.

Table 7. *Cross-Case Analysis Across Conceptual Understanding Categories*

Indicators	High-Level Student (AJ)	Medium-Level Student (DD)	Low-Level Student (WW)
Restating concepts	Demonstrated accurate and comprehensive explanation of known and unknown information	Identified basic information but explanations were incomplete	Unable to identify or explain relevant information systematically
Classification of mathematical objects	Correctly distinguished variables, coefficients, and constants and explained relationships among them	Recognized mathematical elements but provided limited explanations regarding their functions	Failed to distinguish mathematical elements appropriately
Examples and non-examples	Generated valid examples and distinguished non-examples based on conceptual characteristics	Did not extend understanding beyond given examples	Unable to generate examples or distinguish non-examples
Multiple mathematical representations	Flexibly transformed contextual situations into symbolic and verbal forms	Constructed symbolic forms but with limited interpretation	Relied on isolated symbolic manipulation without meaningful explanation

The findings reveal a progressive pattern of conceptual understanding across participant categories. Students within the high category demonstrated integrated conceptual structures in which all indicators appeared consistently across written responses and interview explanations. Their understanding extended beyond procedural correctness because they were able to explain the relationships among mathematical elements and justify their reasoning processes. Furthermore, these students demonstrated representational flexibility by moving between contextual information, symbolic expressions, and verbal interpretations without difficulty. In contrast, students within the medium category demonstrated transitional characteristics of conceptual understanding. Although these students successfully identified essential information and constructed mathematical equations, their understanding remained largely dependent on procedural familiarity. Their responses indicated that mathematical procedures were often applied correctly; however, conceptual explanations underlying those procedures were incomplete. The absence of examples and non-examples further suggests limited conceptual generalization, indicating that students had not fully internalized the defining characteristics of linear equations in one variable.

Different characteristics were observed among students in the low category. Their responses frequently demonstrated direct symbolic manipulation without meaningful interpretation of mathematical relationships. Variables and symbols were used mechanically and appeared to function as procedural tools rather than representations of contextual information. Interview findings reinforced these observations, indicating that symbol use was primarily based on memorization and repetitive exposure rather than conceptual reasoning. Consequently, students in this category demonstrated fragmented conceptual structures and limited representational awareness. The cross-case findings also indicate that conceptual understanding develops progressively rather than independently across indicators. Students who demonstrated strong performance in restating concepts also tended to perform well in classifying mathematical objects and constructing multiple representations. Conversely, students who experienced difficulties in identifying conceptual relationships frequently encountered problems in generating examples and interpreting symbolic forms. This pattern suggests that conceptual understanding is not composed of isolated skills but represents an interconnected cognitive structure in which weaknesses in one dimension may influence performance in other dimensions. Cross-case analysis demonstrates that differences among participant categories were associated not only with procedural performance but also with the depth and coherence of conceptual structures underlying mathematical thinking. High-level students demonstrated integrated conceptual reasoning, medium-level students displayed transitional conceptual characteristics, and low-level students relied predominantly on symbolic procedures without meaningful conceptual interpretation.

DISCUSSION

The findings of this study indicate that students' conceptual understanding of linear equations in one variable developed progressively across different levels of ability rather than appearing as isolated differences in achievement. Cross-case analysis demonstrated that students in the high category exhibited integrated conceptual structures across all indicators. In contrast, medium-level students displayed transitional characteristics and low-level students demonstrated fragmented understanding. This pattern suggests that conceptual understanding should be interpreted as a developmental cognitive process in which mathematical ideas become increasingly interconnected through learning experiences. The present findings support the proposition of Rittle-Johnson et al. (2001) that conceptual understanding and procedural knowledge develop through an iterative process in which conceptual insights continuously influence procedural refinement and vice versa. Likewise, Spinner and Fraser (2005) argued that conceptual growth occurs when learners progressively reorganize cognitive structures through meaningful learning experiences. From this perspective, differences among students in the present study do not merely reflect variations in performance outcomes but represent differences in the organization and integration of mathematical knowledge.

The findings further suggest that conceptual understanding extends beyond procedural fluency and requires learners to establish meaningful relationships among mathematical ideas. High-level students consistently demonstrated flexibility in moving between contextual information, symbolic representations, and verbal explanations, whereas medium- and low-level students frequently relied on procedural operations without explaining the underlying relationships among mathematical elements. Such findings align with Greeno (1978), who argued that procedural knowledge alone does not necessarily indicate conceptual understanding because understanding emerges when learners recognize relationships among mathematical structures. Similarly, Greeno (2017) emphasized that mathematical understanding involves meaningful interpretation of relationships among ideas rather than mechanical application of procedures. In the present study, medium- and low-level students appeared to depend largely on procedural familiarity, whereas high-level students demonstrated characteristics of relational understanding by explaining why mathematical procedures were applied and how they were connected to the contextual situation.

One of the most notable findings concerns students' difficulties in generating examples and non-examples. Students in the high category successfully generated additional examples and distinguished non-examples according to conceptual characteristics, whereas medium- and low-level students struggled to move beyond examples explicitly presented in the problem context. This finding indicates limitations in conceptual generalization and suggests that students' knowledge often remained dependent on specific procedural situations. According to Vergnaud (2016), mathematical concepts involve networks of relationships and situations rather than isolated definitions; therefore, conceptual understanding requires learners to recognize the defining characteristics that distinguish one concept from another. Likewise, Vinner (2011) emphasized that examples play a critical role in conceptual development because they shape learners' conceptual images and influence how mathematical ideas are interpreted. Research by Watson and Mason (2002) further demonstrated that generating examples encourages learners to construct and refine conceptual relationships, while Cohen and Carpenter (1980) showed that exposure to non-examples helps learners identify conceptual boundaries. Consequently, the inability of medium- and low-level students to generate examples and non-examples suggests incomplete conceptual structures that limited their ability to extend mathematical understanding beyond procedural contexts.

Another important finding relates to students' representational abilities. Students in the high category demonstrated flexibility in transforming contextual situations into symbolic forms and verbal interpretations, whereas medium- and low-level students frequently used symbols mechanically without understanding their representational meaning. Interview findings reinforced this interpretation, particularly among low-level students who reported using symbols such as x simply because they frequently encountered them in mathematics problems. These findings suggest that symbols functioned primarily as procedural tools rather than meaningful representations of mathematical relationships. Such patterns are consistent with Acevedo Nistal et al. (2009), who conceptualized representational flexibility as the ability to move appropriately among different forms of representation according to contextual demands. Similarly, Greer (2009) argued that representational flexibility constitutes a critical characteristic of mathematical expertise because it enables learners to interpret and connect mathematical ideas effectively. Furthermore, Dreher et al. (2016) suggested that multiple representations facilitate deeper conceptual understanding because they support learners in constructing relationships among mathematical ideas. Therefore, difficulties observed among medium- and low-level students may indicate insufficient representational awareness rather than purely computational limitations.

The findings additionally reveal that conceptual understanding indicators did not operate independently but instead appeared as interconnected dimensions of mathematical thinking. Students who demonstrated strong performance in restating concepts also tended to exhibit stronger performance in classification, example generation, and representation tasks. Conversely, students experiencing difficulties in one indicator frequently demonstrated weaknesses across other dimensions. Such patterns support the view that conceptual understanding represents an interconnected cognitive structure rather than a collection of isolated skills. Geeslin and Shavelson (1975) argued that students organize mathematical knowledge within cognitive structures that influence how new information is interpreted and integrated. Similarly, Yang et al. (2018) found that students with stronger mathematical cognitive structures demonstrated more systematic relationships among mathematical ideas. Therefore, weaknesses observed among low-level students may not represent isolated deficiencies but rather indicate broader limitations in the organization of conceptual knowledge.

From a pedagogical perspective, these findings imply that mathematics instruction should move beyond emphasizing procedural repetition and computational accuracy. Learning environments should encourage students to construct relationships among concepts, generate examples and non-examples, and represent mathematical ideas through multiple forms. Contextual learning approaches may provide effective opportunities for supporting conceptual development because they require learners to interpret mathematical ideas within meaningful situations. Previous studies have shown that contextual learning environments improve conceptual understanding by encouraging learners to connect mathematical concepts with authentic experiences (Herawaty & Widada, 2017; Yudha et al., 2019). Furthermore, differentiated instructional support appears necessary because students demonstrated distinct conceptual characteristics across ability levels. Students with lower conceptual understanding may require systematic scaffolding processes that gradually support conceptual construction. According to Anghileri (2006) and van Oers (2020), scaffolding strategies support students by providing temporary guidance that assists learners in progressing toward higher levels of understanding. Consequently, adaptive instructional approaches may be necessary to facilitate conceptual development among students with different learning characteristics.

Overall, the findings of this study suggest that conceptual understanding in mathematics is characterized by progressive and interconnected cognitive structures rather than isolated procedural abilities. The study further demonstrates that meaningful mathematical learning requires students not only to know how procedures are applied but also to understand why such procedures operate and how they are related to broader conceptual systems.

CONCLUSIONS

This study examined students' conceptual understanding in solving contextual problems related to linear equations in one variable through four indicators: restating concepts, classifying mathematical objects, generating examples and non-examples, and representing concepts using multiple mathematical forms. The findings indicate that students' conceptual understanding is characterized by progressive differences in cognitive organization rather than merely differences in procedural performance. High-level students demonstrated integrated conceptual structures reflected in coherent reasoning, representational flexibility, and meaningful interpretation of mathematical relationships. In contrast, medium-level students exhibited transitional characteristics in which procedural competence was present but conceptual explanations remained limited, whereas low-level students demonstrated fragmented understanding characterized by symbolic dependence and procedural imitation. The study further suggests that conceptual understanding operates as an interconnected cognitive structure in which conceptual indicators influence one another rather than functioning independently. Difficulties in recognizing conceptual relationships were associated with weaknesses in generating examples, interpreting representations, and applying mathematical ideas across contextual situations. These findings indicate that students' conceptual understanding extends beyond the ability to obtain correct answers and involves the construction of meaningful relationships among mathematical concepts and representations. From a theoretical perspective, this study contributes to the understanding of conceptual development in mathematics by demonstrating that conceptual understanding emerges progressively through interconnected dimensions of mathematical thinking. From a practical perspective, the findings highlight the importance of designing mathematics instruction that emphasizes conceptual exploration, multiple representations, contextual learning experiences, and structured scaffolding processes to support learners with different levels of understanding. Several limitations should be acknowledged. The study involved participants from a single classroom context and focused exclusively on linear equations in one variable. In addition, the investigation employed a qualitative design that emphasized conceptual characteristics rather than examining instructional effects over time. Future studies are therefore recommended to involve larger and more diverse participant groups, investigate different mathematical topics, and employ longitudinal or intervention-based approaches to examine how conceptual understanding develops across broader instructional contexts.

ADDITIONAL INFORMATION

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Conflict of Interest	The authors declare that there are no competing interests that could have influenced the outcomes of this research.
Data Availability	The data supporting the findings of this study are available from the corresponding author upon reasonable request.
Author Contributions	**R:** Conceptualization, investigation, data collection, formal analysis, interpretation of findings, and manuscript preparation. **S S:** Supervision, methodological guidance, manuscript review, editing, and refinement of the final manuscript. All authors have read and approved the final version of the manuscript.

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