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Mathematical Critical Thinking Profiles of Seventh-Grade Students in Solving Fraction Problems within Realistic Mathematics Education Contexts

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Abstract

This study investigates seventh-grade students' mathematical critical thinking profiles in solving fraction problems involving different denominators within Realistic Mathematics Education (RME) contexts. A qualitative multiple-case descriptive design was employed involving 18 students who completed contextual essay-based mathematical tasks, with six students purposively selected for in-depth semi-structured interviews representing high-, medium-, and low-ability categories. Data were collected through written tasks and interview protocols developed according to Ennis's five critical thinking indicators: interpretation, analysis, evaluation, inference, and explanation. Data analysis followed an interactive qualitative approach involving data reduction, data display, and conclusion verification. The findings revealed substantial variation in students' mathematical critical thinking across indicators and ability categories. High-level students demonstrated relatively systematic and coherent reasoning processes, whereas medium- and low-level students exhibited fragmented reasoning characterized by procedural uncertainty and conceptual difficulties. Interpretation emerged as the most accessible indicator, while inference and explanation represented the most challenging dimensions. Additive misconception was identified as the most dominant conceptual difficulty, particularly among low-level students, indicating broader weaknesses in fraction understanding rather than isolated procedural errors. Furthermore, the findings suggest that mathematical critical thinking indicators function as interconnected dimensions, where difficulties occurring during earlier reasoning stages frequently coincided with limitations in subsequent processes. This study contributes to mathematics education literature by providing a multidimensional understanding of students' mathematical critical thinking through the integration of written responses and interview data. The findings highlight the importance of instructional practices emphasizing conceptual understanding, reflective reasoning, and meaningful mathematical contexts.

INTRODUCTION

Mathematics education at the lower secondary level plays a fundamental role in fostering higher-order thinking skills, particularly mathematical critical thinking. Mathematics should not merely function as a procedural activity involving symbolic manipulation and numerical computation; rather, it should facilitate students' capacity to interpret information, analyze relationships, evaluate arguments, formulate conclusions, and justify reasoning logically. Mathematical critical thinking is considered an essential component of mathematics learning because it enables students to construct meaningful understanding and engage in reflective reasoning during problem-solving activities (Jablonka, 2020; Aldila Afriansyah et al., 2021). In contemporary mathematics education, the development of critical thinking has become increasingly important because students are expected not only to obtain correct answers but also to explain reasoning processes and make informed decisions based on mathematical evidence. Critical thinking therefore functions as a cognitive process that supports conceptual understanding and problem-solving competence. However, empirical findings continue to indicate that students experience difficulties in demonstrating analytical and evaluative reasoning abilities in mathematics learning contexts (Said & Lukmana, 2020). These findings suggest that mathematical critical thinking remains insufficiently developed and continues to present a significant challenge in classroom practice.

The importance of mathematical critical thinking becomes particularly evident in fraction learning because fractions involve cognitive processes that extend beyond procedural calculations and require students to coordinate multiple forms of reasoning. Understanding fractions demands conceptual comprehension of part-whole relationships, proportional reasoning, magnitude comparison, and equivalence transformations (Lamon, 2012). Unlike whole numbers, fractions require students to mentally reconstruct relationships among quantities while simultaneously interpreting symbolic representations and contextual situations. Therefore, solving fraction problems—particularly those involving different denominators—requires students not only to apply computational procedures but also to interpret information, analyze quantitative relationships, evaluate solution strategies, and justify conclusions logically. Nevertheless, many students continue to experience misconceptions in fraction operations and rely predominantly on procedural approaches without conceptual understanding. Whole-number bias frequently causes students to transfer inappropriate reasoning patterns from whole-number operations into fraction contexts, leading to systematic conceptual errors (Ni & Zhou, 2005). Furthermore, difficulties in fraction understanding often persist over time and may influence subsequent mathematical achievement and broader mathematical development (Ye et al., 2016; Jordan et al., 2017). These findings indicate that students' difficulties in fraction learning may also reflect limitations in mathematical critical thinking processes.

Such challenges may be associated with instructional practices that continue to emphasize procedural learning and teacher-centered approaches. Mathematics instruction frequently prioritizes formula memorization and repetitive exercises while providing limited opportunities for students to construct understanding actively through reasoning and exploration. Under these conditions, students often become accustomed to reproducing algorithms rather than developing conceptual understanding of mathematical relationships. Consequently, students may demonstrate proficiency in executing procedures while simultaneously experiencing difficulties in interpreting problems, evaluating strategies, and explaining the rationale underlying their answers. Such instructional conditions potentially limit opportunities for the development of higher-order thinking skills because learning activities may emphasize answer production rather than reasoning processes (Tambunan & Naibaho, 2019). Consequently, mathematics learning environments should provide opportunities for students to engage actively in contextual reasoning and conceptual construction. One instructional approach that has been proposed to address these limitations is Realistic Mathematics Education (RME), which emphasizes meaningful contexts and supports students in constructing mathematical ideas through active exploration and interaction (Streefland, 2012).

Previous studies have reported that RME contributes positively to various dimensions of students' mathematical learning outcomes, including critical thinking, reasoning abilities, and problem-solving performance. Research findings indicate that contextual mathematical experiences may encourage students to engage more actively in interpretation, analysis, and reflective reasoning processes (Cahyaningsih & Nahdi, 2021; Yohannes & Chen, 2024). Similarly, Amir et al. (2024) found that realistic mathematical learning environments support the development of problem-solving and critical thinking skills through meaningful learning experiences. In addition, RME-based instructional materials have been reported to facilitate the development of higher-order thinking abilities and broader mathematical competencies (Sutarni et al., 2024). These findings suggest that contextual learning environments can potentially support the development of mathematical thinking processes beyond procedural competence. However, despite consistent evidence regarding the effectiveness of RME, existing studies have predominantly focused on overall performance improvement rather than examining the specific dimensions through which students demonstrate critical thinking during mathematical problem-solving processes.

This limitation indicates the existence of an important research gap. Existing studies largely provide evidence regarding improvements in students' achievement outcomes and general critical thinking performance while offering limited understanding of how distinct critical thinking indicators emerge during mathematical problem solving, particularly in fraction topics involving different denominators. Mathematical critical thinking should not be viewed as a single unified construct because it consists of multiple cognitive dimensions that may develop differently across learners and mathematical contexts. Consequently, examining students' critical thinking solely through performance scores may obscure important differences in the reasoning processes underlying students' responses. Moreover, studies integrating written problem-solving tasks with semi-structured interviews to investigate students' cognitive processes remain relatively limited. Therefore, there is still insufficient evidence regarding how students enact interpretation, analysis, evaluation, inference, and explanation processes when solving contextual fraction problems.

The contribution of this study lies in its multidimensional investigation of students' mathematical critical thinking through the examination of five indicators, namely interpretation, analysis, evaluation, inference, and explanation. Rather than focusing exclusively on achievement outcomes, this study investigates how critical thinking processes are enacted during mathematical problem solving through the integration of written responses and semi-structured interview data. Preliminary observations conducted at SMP Negeri 7 Ngabang indicated that seventh-grade students continue to experience difficulties in solving fraction problems involving different denominators, frequently demonstrating computational errors and difficulties in explaining the reasoning underlying their solutions. Therefore, this study addresses the following research question: *How do students demonstrate mathematical critical thinking across interpretation, analysis, evaluation, inference, and explanation when solving fraction problems involving different denominators within Realistic Mathematics Education contexts?* This study is expected to provide a more comprehensive understanding of students' reasoning processes and contribute theoretically and practically to broader discussions regarding mathematical critical thinking in mathematics education.

METHODS

Research Design

This study employed a qualitative multiple-case descriptive design to investigate students' mathematical critical thinking profiles in solving fraction problems involving different denominators within a Realistic Mathematics Education (RME) context. A qualitative approach was considered appropriate because the study aimed to obtain an in-depth understanding of students' reasoning processes rather than to test hypotheses or generate statistical generalizations. Qualitative approaches in mathematics education are particularly useful for examining students' cognitive processes and understanding how mathematical meaning is constructed in natural learning settings (Bikner-Ahsbahs et al., 2015). Furthermore, multiple-case designs facilitate the exploration of similarities and differences across participants and provide opportunities for detailed analysis of students' thinking patterns in different contexts (Yilmaz & Kostur, 2021; Schutte et al., 2026). Therefore, this design was considered appropriate for investigating how students demonstrated critical thinking through interpretation, analysis, evaluation, inference, and explanation during mathematical problem-solving activities.

Research Setting and Participants

The study was conducted at SMP Negeri 7 Ngabang, located in Landak Regency, West Kalimantan, Indonesia. The research setting was selected based on preliminary classroom observations indicating that students experienced difficulties in solving fraction problems involving different denominators. Students frequently demonstrated computational errors and encountered difficulties in explaining the reasoning underlying their solutions, suggesting potential limitations in mathematical critical thinking processes. Participants were selected using purposive sampling based on criteria aligned with the research objectives. Purposive sampling enables researchers to select information-rich participants capable of providing meaningful insights into the phenomenon under investigation (Ames et al., 2019; Ahmad & Wilkins, 2025). Initially, all seventh-grade students completed contextual mathematical tasks designed according to RME principles. Students' responses were analyzed and classified into high-, medium-, and low-ability categories according to their mathematical critical thinking performance. Subsequently, two students from each category were selected as focal participants for in-depth investigation, resulting in six primary participants. The selection of six participants was intended to maximize depth and richness of information rather than statistical representativeness.

Research Instruments

The researcher functioned as the primary instrument responsible for designing the study, collecting data, analyzing findings, interpreting results, and drawing conclusions. Supporting instruments included observation sheets, contextual essay-based mathematical tasks, a mathematical critical thinking assessment rubric, and semi-structured interview guidelines. The essay-based tasks consisted of two contextual problems involving fraction addition and subtraction with different denominators situated within realistic situations relevant to students' daily experiences. Essay-based assessments were selected because higher-level mathematical thinking and critical thinking processes are more effectively captured through tasks requiring explanation and justification rather than simple procedural responses (Koh et al., 2025). Item development was guided by five critical thinking indicators, namely interpretation, analysis, evaluation, inference, and explanation, which were systematically aligned with problem characteristics and expected student responses. Similar approaches have been employed in studies investigating students' critical thinking processes in mathematics learning environments (Zuhaida et al., 2022). Prior to implementation, the instruments underwent expert review to ensure content relevance, clarity, and alignment with intended constructs. Content validity evidence was established based on expert judgments following systematic validation procedures such as Aiken's V-based approaches used in mathematics assessment development (Kania et al., 2024). Revisions were conducted according to experts' feedback to improve instrument appropriateness and comprehensibility. A mathematical critical thinking rubric was used to classify students' performance into high, medium, and low categories across each indicator. The rubric functioned as a qualitative analytical framework to facilitate systematic interpretation of students' thinking profiles. Scoring procedures were used exclusively for participant classification and descriptive purposes and were not intended for inferential statistical analysis.

Data Collection Procedures

Data collection was conducted through classroom observation, written tasks, semi-structured interviews, and documentation. Observation was initially conducted to identify classroom conditions, students' learning characteristics, and instructional practices related to fraction learning. Subsequently, students completed contextual mathematical tasks designed according to RME principles. Students' written responses were then analyzed to identify characteristics of mathematical critical thinking and classify participants according to ability levels. Semi-structured interviews were conducted individually with selected participants following analysis of written responses. Interviews were intended to explore students' reasoning processes more deeply and clarify information that could not be fully identified from written responses alone. Flexible questioning procedures were employed to encourage participants to explain their reasoning naturally and comprehensively. Such approaches are particularly useful for exploring students' mathematical thinking processes beyond observable written responses (Simpson & Haltiwanger, 2017). Documentation, including students' written work, field notes, and photographs of learning activities, was also collected to support data interpretation.

Data Analysis

Data analysis followed the interactive model proposed by Miles et al. (2014), consisting of data reduction, data display, and conclusion drawing and verification. Data reduction involved organizing and coding written responses and interview transcripts according to the mathematical critical thinking indicators. Data reduction involved organizing, coding, and categorizing students' written responses and interview transcripts according to the five mathematical critical thinking indicators: interpretation, analysis, evaluation, inference, and explanation. Data display was conducted through descriptive narratives, tables, and excerpts from students' responses to identify patterns and relationships among critical thinking indicators. Subsequently, conclusions were developed iteratively through continuous comparison between written responses and interview findings. The analysis process involved repeated examination of the data to ensure that interpretations accurately reflected students' actual reasoning processes.

Trustworthiness

Several procedures were implemented to ensure the trustworthiness of the findings. Trustworthiness in qualitative research is essential to ensure that interpretations accurately represent the investigated phenomenon and maintain methodological rigor throughout the research process (Williams & Morrow, 2009). Source triangulation was conducted by comparing information obtained from written responses, interviews, classroom observations, and supporting documentation to ensure consistency across multiple sources of evidence. Member checking was also employed by confirming interview interpretations with participants to ensure that the findings accurately represented participants' intended meanings. In addition, peer debriefing was conducted through discussions with mathematics education colleagues to minimize researcher bias and strengthen analytical consistency. An audit trail documenting research procedures, coding decisions, and analytical processes was maintained throughout the study to improve transparency and methodological rigor. These procedures were implemented to strengthen the credibility, dependability, and confirmability of the findings.

RESULT

This study examined seventh-grade students' mathematical critical thinking in solving fraction addition and subtraction problems involving different denominators within the context of Realistic Mathematics Education (RME). The analysis focused on five critical thinking indicators: interpretation, analysis, evaluation, inference, and explanation. Data were obtained through contextual written tasks and semi-structured interviews involving six focal participants representing high- (S1–S2), medium- (S3–S4), and low-ability groups (S5–S6). Overall, the findings indicate substantial variation in students' mathematical critical thinking across indicators and ability categories. Students in the high category demonstrated relatively complete critical thinking processes, whereas students in the medium and low categories exhibited incomplete reasoning patterns and conceptual difficulties during problem solving.

Distribution of Students' Mathematical Critical Thinking Levels

The distribution of students' mathematical critical thinking levels indicated that the majority of students were classified within the low and very low categories. In contrast, relatively few students achieved high and very high levels. Out of 18 students, most demonstrated limited performance across several critical thinking indicators. Written responses and interview findings further revealed that students frequently encountered difficulties when solving fraction problems involving different denominators. A recurrent pattern identified across student responses involved additive misconceptions, in which students directly added numerators and denominators without transforming fractions into equivalent forms through common denominators. This misconception appeared consistently among students classified within the low-ability group and emerged as the most prominent conceptual difficulty identified in this study.

Critical Thinking Patterns Across Ability Levels

Cross-case analysis revealed distinct patterns of mathematical critical thinking across ability categories. High-level students demonstrated systematic and connected reasoning processes across indicators. Medium-level students generally showed partial understanding but exhibited inconsistency in applying concepts and justifying procedures. In contrast, low-level students demonstrated fragmented reasoning characterized by procedural errors, conceptual misconceptions, and limited ability to explain problem-solving processes.

Interpretation

Students' interpretation abilities differed across ability categories, particularly in identifying relevant information and understanding contextual requirements within fraction problems. Cross-case analysis revealed progressive differences in how students recognized given information, interpreted problem requirements, and connected contextual meaning to mathematical procedures.

High-level students (S1–S2) demonstrated complete identification of problem information. n written responses, S1 correctly identified all known and required information from the contextual task and translated it into mathematical expressions before proceeding with calculations. The student explicitly identified the fraction quantities involved and recognized that denominator equalization was required before performing operations. Figure 1 presents a representative response from S1 illustrating a complete interpretation process.

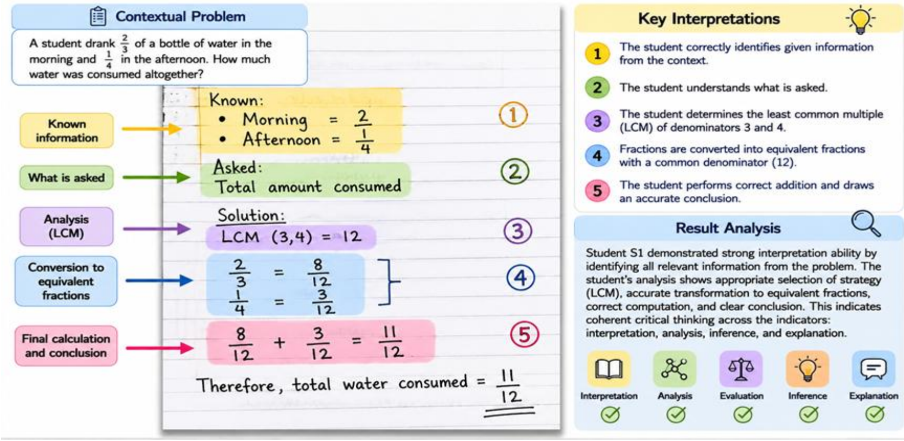

Figure 1. *High-Level Student (S1): Identification and Interpretation of Contextual Information in Fraction Problem Solving*

Figure 1 illustrates how the student identified known information, recognized the mathematical requirement of denominator equalization, transformed fractions into equivalent forms, and produced an appropriate conclusion. Interview data further supported this pattern:

Researcher: How did you determine what information was important in this problem?

S1: I first looked at the fractions given and identified what was asked. Since the denominators were different, I knew they needed to be made equal before adding them.

This response indicates that high-level students not only extracted information but also connected contextual information to mathematical requirements. Their written work and verbal explanations demonstrated coherent transitions from contextual understanding toward mathematical representation. Medium-level students (S3–S4) identified the primary information but occasionally omitted certain problem components. Written responses showed that these students recognized numerical values correctly and were generally able to apply solution procedures; however, they did not consistently identify all contextual elements before initiating calculations. Figure 2 presents a representative response from S3 illustrating partial interpretation processes.

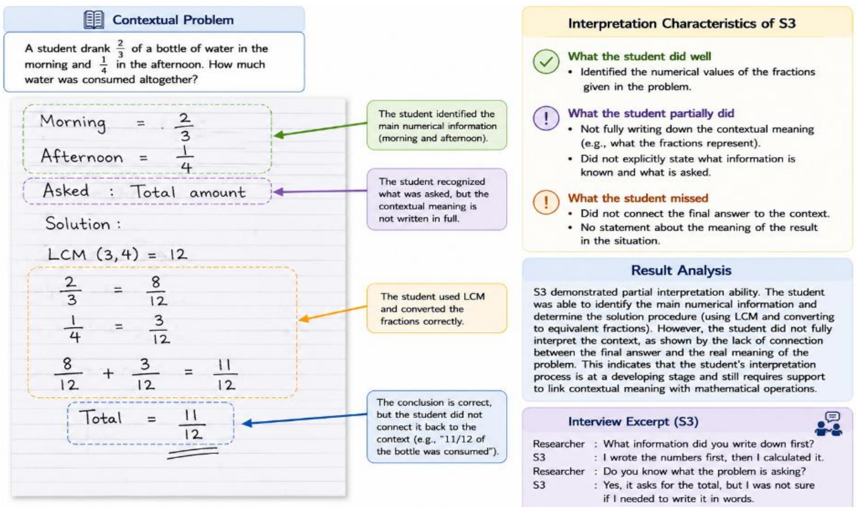

Figure 2. *Medium-Level Student (S3): Partial Interpretation of Contextual Information in Fraction Problem Solving*

Figure 2 shows that the student successfully identified the main numerical information and applied denominator equalization procedures, but did not explicitly connect the mathematical process with contextual meaning. Interview findings further supported this pattern:

Researcher: What information did you write down first?

S3: I wrote the numbers first, then I calculated them. I was not sure whether I needed to explain it in words.

This finding suggests that medium-level students demonstrated partial interpretation processes in which procedural understanding existed but contextual understanding remained incomplete. Low-level students (S5–S6) demonstrated substantial difficulties during interpretation. Written responses revealed that these students focused primarily on visible numerical symbols without identifying problem requirements or contextual meaning. Rather than organizing known and required information, students directly performed computational procedures. Figure 3 presents a representative response from S5 illustrating difficulties in interpretation.

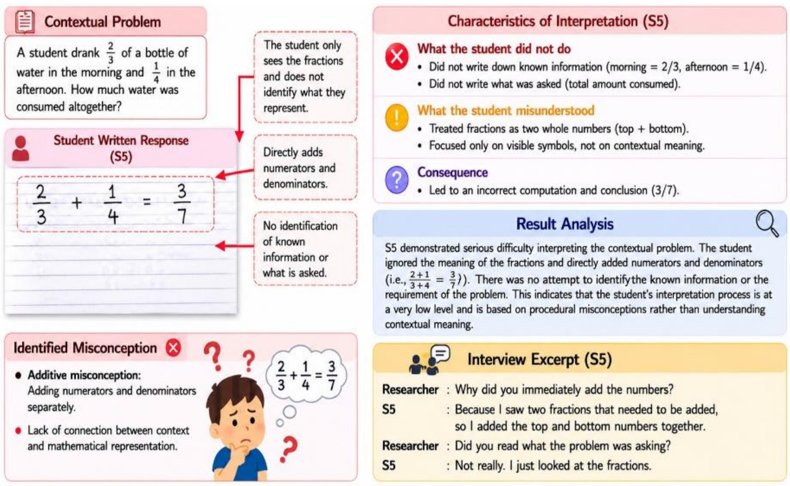

Figure 3. *Low-Level Student (S5): Difficulty in Interpreting Contextual Information*

Figure 3 illustrates that the student immediately performed numerical operations without identifying known information, determining what was asked, or recognizing the contextual meaning embedded in the problem. Interview findings further revealed conceptual misunderstanding:

Researcher: Why did you immediately add the numbers?

S5: Because I saw two fractions that needed to be added, so I added the top and bottom numbers together.

This finding indicates that low-level students relied primarily on symbolic procedures rather than contextual interpretation. The response also demonstrates the presence of additive misconceptions, where fractions were treated as separate whole numbers rather than integrated numerical representations. Overall, cross-case comparison indicates that interpretation ability progressively decreased across categories. High-level students demonstrated complete contextual interpretation and procedural planning, medium-level students exhibited partial interpretation characterized by incomplete contextual connections, whereas low-level students relied primarily on visible symbols and demonstrated substantial conceptual misunderstanding.

Analysis

Differences were also observed in students' analytical processes, particularly in identifying relationships among fractions and determining appropriate solution strategies. Cross-case analysis revealed distinct analytical patterns across ability categories. High-level students demonstrated systematic reasoning processes, whereas medium-level students showed partial procedural understanding with some uncertainty. In contrast, low-level students experienced considerable difficulties in analyzing relationships among fractions and frequently relied on inappropriate procedures. High-level students (S1–S2) demonstrated systematic analytical processes by identifying mathematical relationships and selecting appropriate strategies before performing calculations. In written responses, S1 first determined the least common multiple (LCM) of denominators and transformed fractions into equivalent forms prior to conducting operations. The student consistently connected contextual situations with mathematical representations and demonstrated organized problem-solving procedures. Figure 4 presents a representative response from S1 illustrating systematic analytical processes.

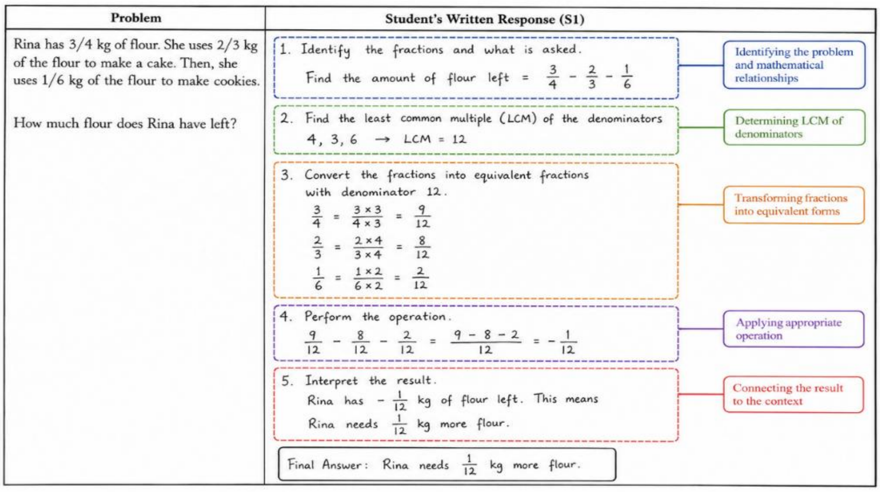

Figure 4. *High-Level Student (S1): Systematic Analysis of Fraction Relationships and Solution Strategy*

Figure 4 illustrates the student's ability to identify relationships among fractions, determine the least common multiple, transform fractions into equivalent forms, and apply an organized solution procedure. Interview findings further supported this pattern:

Researcher: How did you decide what strategy to use?

S1: I saw that the denominators were different, so I looked for the least common multiple first because they could not be added directly.

This response indicates that high-level students analyzed mathematical structures before initiating calculations and demonstrated coherent connections between conceptual understanding and procedural decisions. Medium-level students (S3–S4) demonstrated partial analytical understanding. Written responses showed that students generally recognized the need for denominator equalization and attempted to apply correct procedures. However, these students occasionally showed hesitation when selecting or applying solution strategies. Figure 5 presents a representative response from S3 illustrating partial analytical processes.

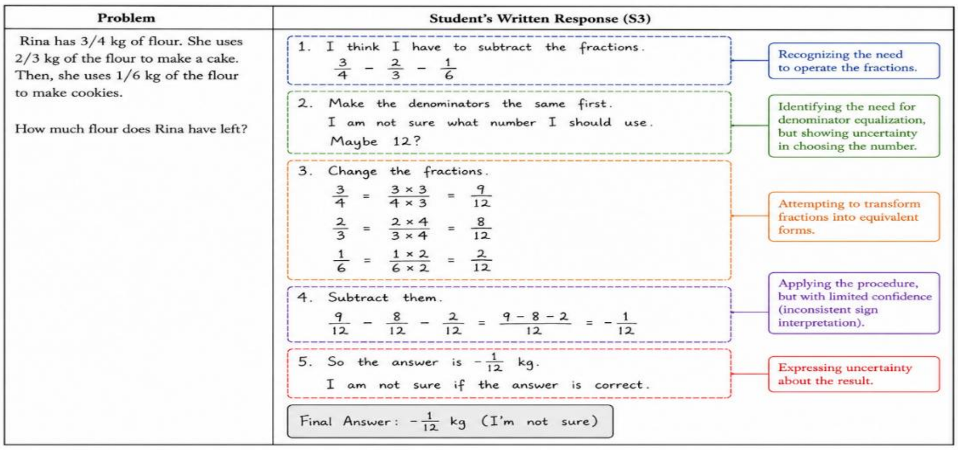

Figure 5. *Medium-Level Student (S3): Partial Analysis of Fraction Relationships*

Figure 5 shows that the student identified relationships among fractions and attempted appropriate procedures, but demonstrated uncertainty and incomplete procedural confidence.

Interview data further supported this finding:

Researcher: Why did you use this procedure?

S3: I remembered that different denominators should become the same first, but sometimes I was not sure whether I used the correct numbers.

This finding suggests that medium-level students possessed partial procedural understanding but lacked complete analytical confidence during problem solving. Low-level students (S5–S6) demonstrated considerable difficulties during analytical processes. Written responses revealed that these students were generally unable to establish relationships among fractions and frequently relied on additive procedures by directly combining numerators and denominators. Figure 6 presents a representative response from S5 illustrating analytical difficulties.

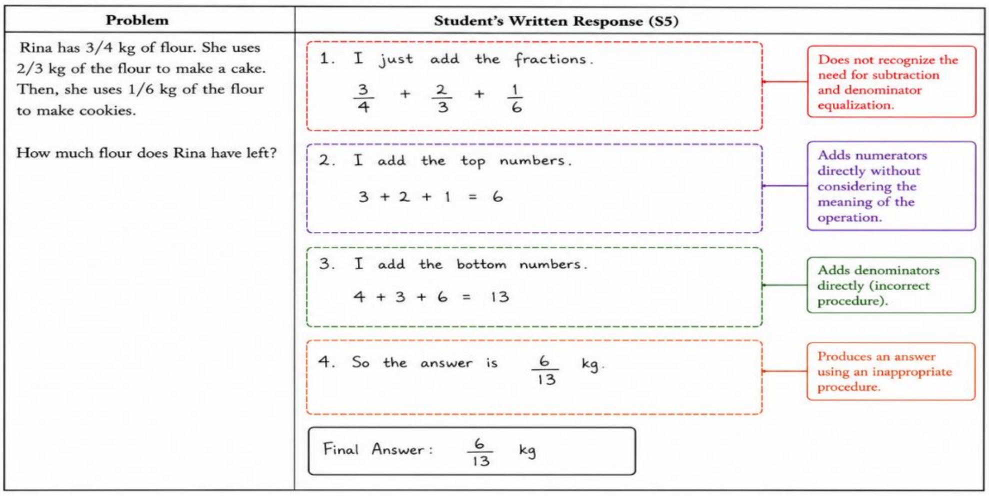

Figure 6. *Low-Level Student (S5): Difficulty in Analyzing Fraction Relationships*

Figure 6 illustrates that the student failed to identify relationships among fractions and directly applied incorrect additive procedures without considering denominator equivalence.

Interview findings further revealed conceptual misunderstanding:

Researcher: Why did you directly add the fractions?

S5: I thought adding fractions meant adding all the numbers together.

This response suggests that low-level students focused on visible numerical symbols rather than analyzing mathematical relationships between fractions. Overall, cross-case comparison indicates that analytical ability progressively decreased across categories. High-level students demonstrated systematic analytical reasoning and strategic selection of procedures; medium-level students exhibited partial analytical understanding accompanied by uncertainty. In contrast, low-level students demonstrated fragmented reasoning characterized by conceptual misconceptions and inappropriate solution strategies.

Evaluation

Students' evaluation abilities demonstrated substantial differences across ability categories, particularly in identifying errors, examining solution procedures, and justifying the appropriateness of mathematical reasoning. Cross-case analysis revealed that students with high mathematical critical thinking abilities demonstrated stronger evaluative processes, whereas students in medium and low categories experienced difficulties in critically examining their responses and validating procedures. High-level students (S1–S2) demonstrated strong evaluation abilities by identifying incorrect procedures and providing conceptual justification for why particular methods were inappropriate. Written responses showed that students not only obtained correct answers but also reviewed their procedures and compared alternative approaches before concluding. Figure 7 presents a representative response from S1 illustrating evaluative processes during fraction problem solving.

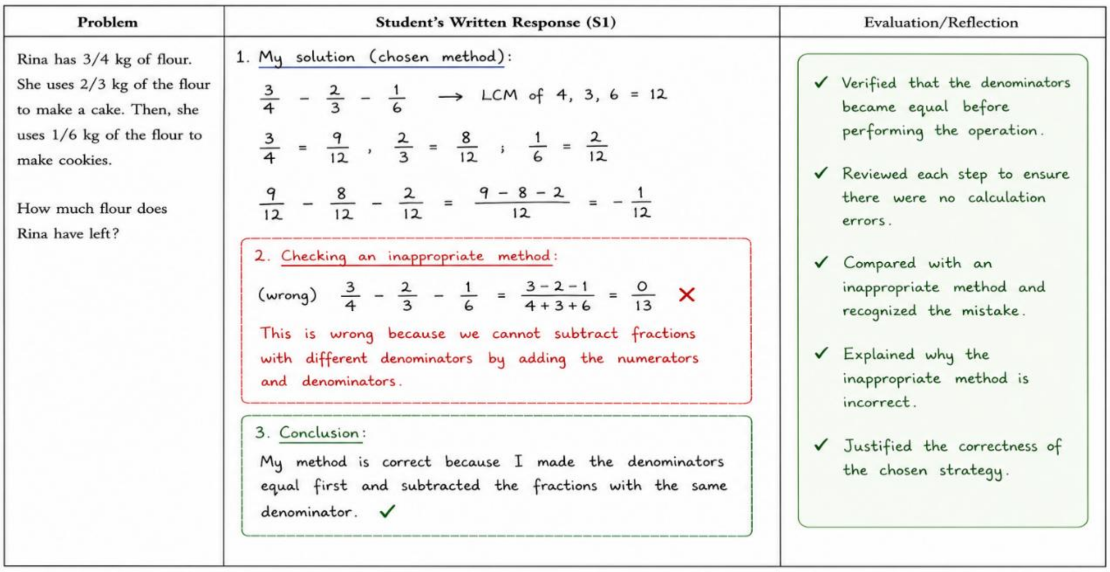

Figure 7. *High-Level Student (S1): Evaluation of Fraction Solution Procedures*

Figure 7 illustrates the student's ability to verify procedures, recognize inappropriate methods, and justify the correctness of the selected strategy.

Interview findings further supported this pattern:

Researcher: How did you know that your solution was correct?

S1: I checked whether the denominators had become equal and looked at each step again to make sure I did not make a mistake.

This response indicates that high-level students actively monitored and verified their own reasoning processes before accepting a final solution. Medium-level students (S3–S4) recognized procedural inconsistencies but provided limited conceptual explanations. Written responses showed that students occasionally identified incorrect steps and attempted revisions; however, their evaluations were frequently based on procedural familiarity rather than conceptual understanding. Figure 8 presents a representative response from S3 illustrating partial evaluative processes.

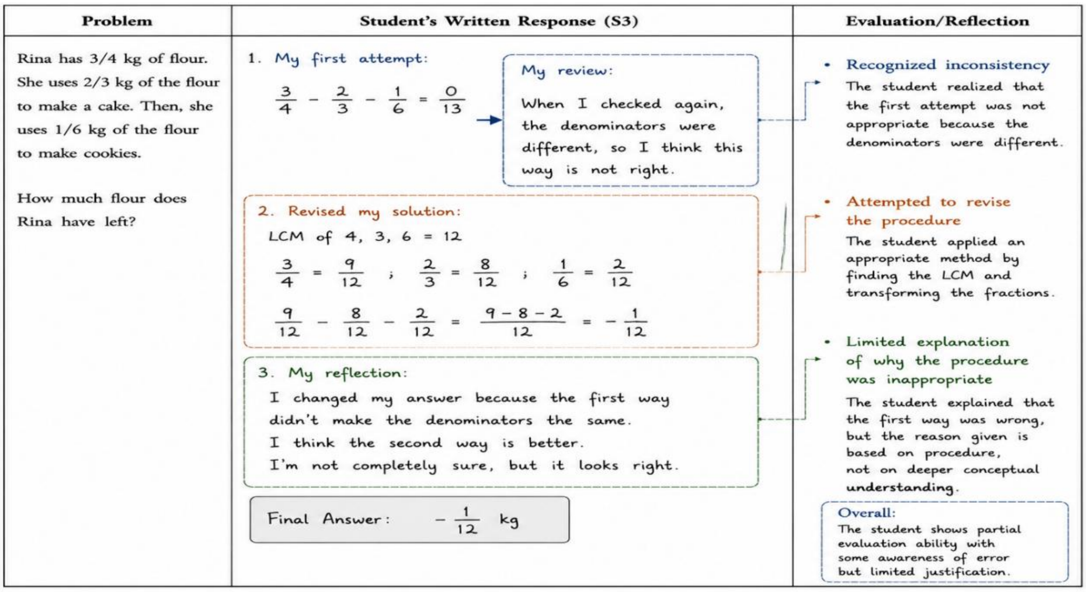

Figure 8. *Medium-Level Student (S3): Partial Evaluation of Solution Procedures*

Figure 8 shows that the student identified procedural inconsistencies but demonstrated limited explanation regarding why the procedure was inappropriate.

Interview findings further supported this observation:

Researcher: Why did you change your answer?

S3: I thought it looked wrong, so I changed it, but I was not completely sure why.

This finding suggests that medium-level students possessed some awareness of procedural errors but lacked confidence and conceptual justification during evaluation processes. Low-level students (S5–S6) demonstrated substantial difficulties in evaluation. Written responses indicated that these students frequently accepted incorrect solutions without reviewing their procedures or considering alternative approaches. Figure 9 presents a representative response from S5 illustrating evaluative difficulties.

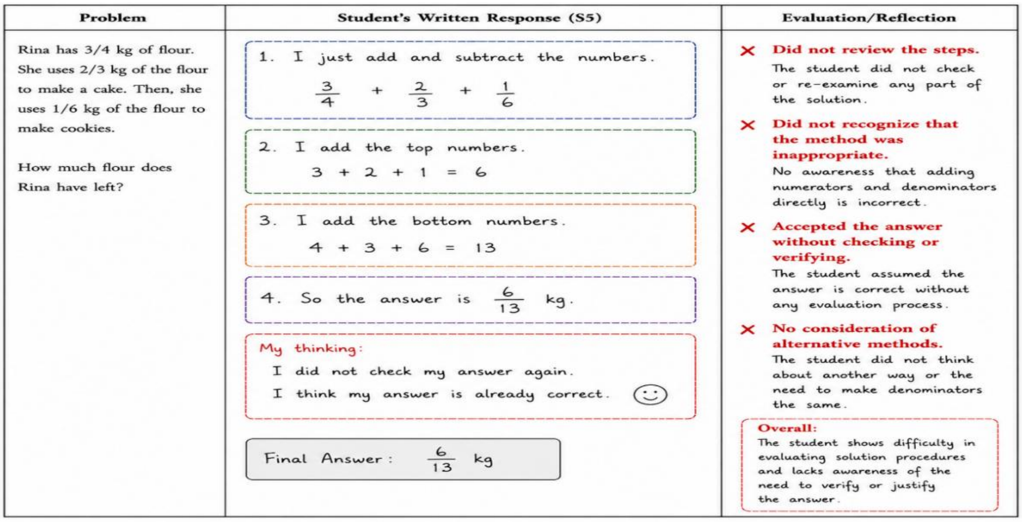

Figure 9. *Low-Level Student (S5): Difficulty in Evaluating Solution Procedures*

Figure 9 illustrates that the student accepted an incorrect procedure without identifying conceptual inconsistencies or reviewing the validity of the solution process.

Interview findings further revealed limitations in evaluative thinking:

Researcher: Did you check your answer after finishing?

S5: No, because I thought the answer was already correct.

This response suggests that low-level students demonstrated limited awareness of the need to verify mathematical procedures and critically examine their own responses. Overall, cross-case comparison indicates that evaluation abilities progressively decreased across categories. High-level students demonstrated systematic monitoring and conceptual justification, and medium-level students exhibited partial evaluative awareness accompanied by uncertainty. In contrast, low-level students demonstrated limited critical examination and frequently accepted incorrect procedures without verification.

Inference

Inference emerged as one of the weakest indicators among medium- and low-level students, particularly in formulating conclusions based on logical reasoning and appropriate mathematical procedures. Cross-case analysis revealed notable differences in how students generated conclusions from the information and procedures they had previously constructed. High-level students demonstrated systematic inferential reasoning, whereas medium-level students exhibited incomplete reasoning processes. In contrast, low-level students frequently generated unsupported conclusions derived from invalid procedures. High-level students (S1–S2) demonstrated strong inferential abilities by formulating conclusions based on systematic comparisons and appropriate mathematical procedures. Written responses showed that students not only obtained correct solutions but also explicitly connected their final results with the contextual meaning of the problem. Conclusions were derived from previous analytical and evaluative processes and were logically consistent with the procedures applied. Figure 10 presents a representative response from S1 illustrating systematic inferential reasoning.

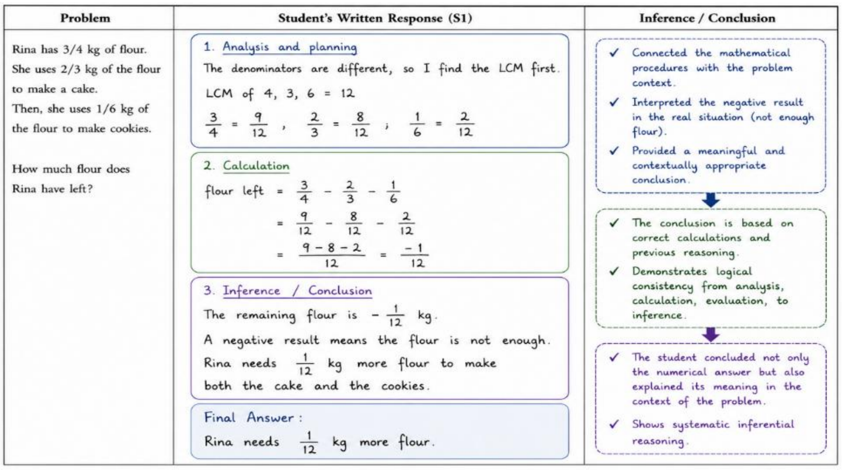

Figure 10. *High-Level Student (S1): Systematic Inference in Fraction Problem Solving*

Figure 10 illustrates the student's ability to formulate conclusions logically by connecting mathematical procedures with contextual meaning and final outcomes.

Interview findings further supported this pattern:

Researcher: How did you decide on your conclusion?

S1: After I finished the calculation, I looked again at what the problem asked and used the result to answer it.

This response indicates that high-level students constructed conclusions through systematic reasoning rather than relying solely on computational outcomes. Medium-level students (S3–S4) frequently generated conclusions based on incomplete reasoning processes. Written responses indicated that students generally reached appropriate numerical results but often provided conclusions without adequately connecting them to the contextual situation. Figure 11 presents a representative response from S3 illustrating partial inferential reasoning.

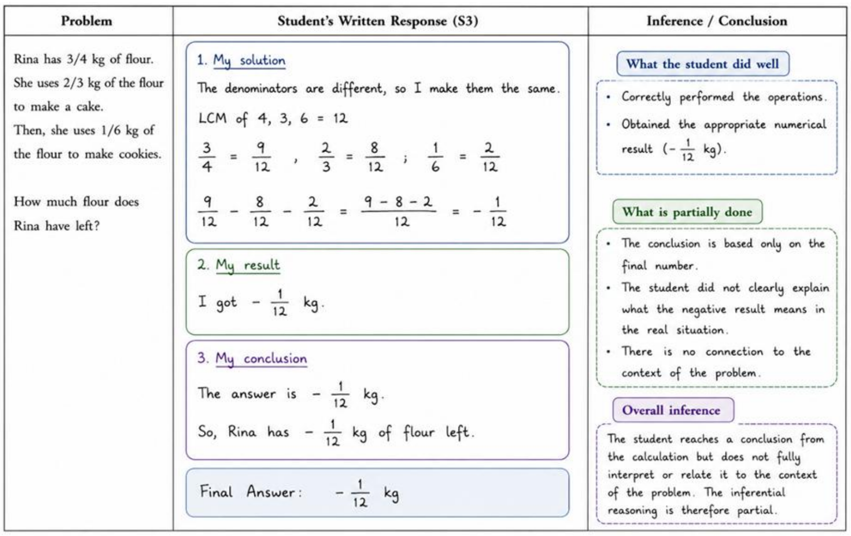

Figure 11. *Medium-Level Student (S3): Partial Inference in Fraction Problem Solving*

Figure 11 shows that the student obtained an appropriate result but demonstrated incomplete reasoning when formulating contextual conclusions.

Interview findings further supported this observation:
Researcher: Why did you write this conclusion?
S3: Because that was the result of my calculation, but I was not sure how to explain it in the problem.

This finding suggests that medium-level students tended to treat conclusions as direct outputs of calculations rather than products of reasoning processes. Low-level students (S5–S6) demonstrated substantial difficulties in inferential reasoning. Written responses revealed that students frequently generated unsupported conclusions or conclusions derived from invalid procedures. In some cases, students wrote numerical results without explaining their meaning, while in other cases, conclusions were directly based on incorrect operations. Figure 12 presents a representative response from S5 illustrating inferential difficulties.

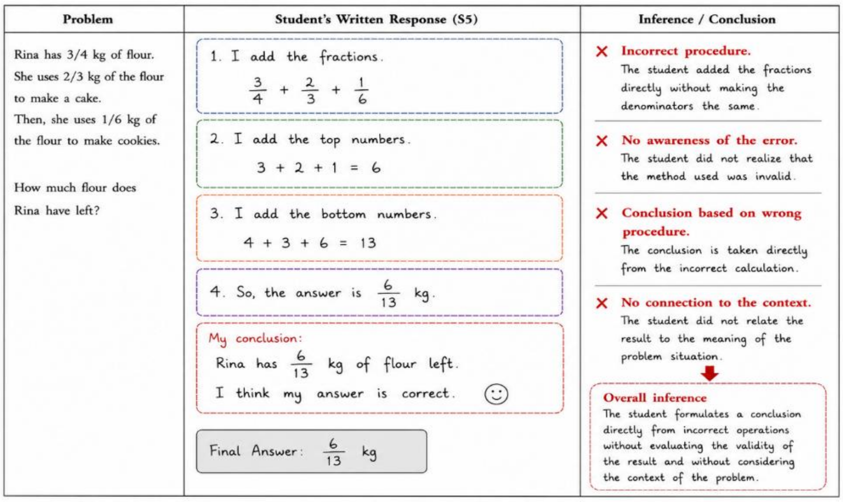

Figure 12. *Low-Level Student (S5): Difficulty in Formulating Conclusions in Fraction Problem Solving*

Figure 12 illustrates that the student generated conclusions directly from incorrect procedures without evaluating the validity of the obtained result.

Interview findings further revealed limitations in inferential reasoning:
Researcher: Why did you think your conclusion was correct?
S5: Because that was the answer I got after adding the numbers.

This response suggests that low-level students relied primarily on computational outcomes without considering whether the conclusion logically reflected the contextual problem. Overall, cross-case comparison indicates that inferential abilities progressively decreased across categories. High-level students demonstrated systematic and contextually meaningful conclusions, and medium-level students exhibited partially developed inferential reasoning. In contrast, low-level students generated unsupported conclusions derived from incomplete or incorrect reasoning processes.

Explanation

Students' explanation abilities reflected substantial differences in conceptual understanding across ability categories, particularly in articulating reasoning processes and communicating mathematical ideas coherently. Cross-case analysis revealed that students with high mathematical critical thinking abilities demonstrated systematic and conceptually grounded explanations, whereas medium-level students showed partial explanatory understanding. In contrast, low-level students experienced considerable difficulty in expressing the reasoning underlying their responses. High-level students (S1–S2) demonstrated strong explanatory abilities by systematically and coherently describing their reasoning processes in both written responses and interviews. Written work indicated that students explained not only procedural steps but also the conceptual basis underlying their decisions. Their explanations connected contextual information, mathematical representations, and final conclusions in a logical sequence. Figure 13 presents a representative response from S1 illustrating systematic explanatory processes.

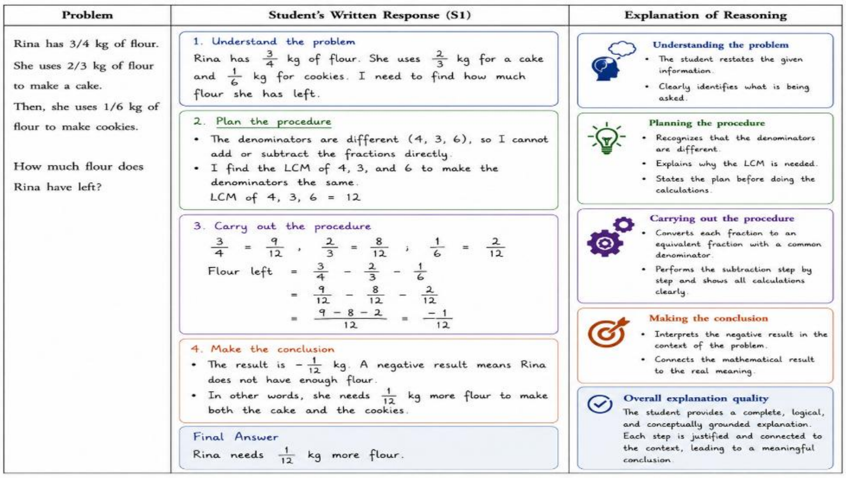

Figure 13. High-Level Student (S1): Systematic Explanation of Mathematical Reasoning in Fraction Problem Solving

Figure 13 illustrates the student's ability to explain mathematical reasoning coherently by connecting contextual understanding, procedural decisions, and conceptual justification.

Interview findings further supported this pattern:

Researcher: Can you explain why you used that procedure?

S1: I used that procedure because the denominators were different, and fractions cannot be added directly unless they represent equal parts. Therefore, I changed them into equivalent fractions first.

This response indicates that high-level students demonstrated conceptual understanding extending beyond procedural knowledge and were able to communicate their reasoning clearly. Medium-level students (S3–S4) described procedural steps but frequently omitted conceptual justification. Written responses showed that students generally explained the sequence of calculations correctly; however, they often failed to explain why specific procedures were applied. Figure 14 presents a representative response from S3 illustrating partial explanatory processes.

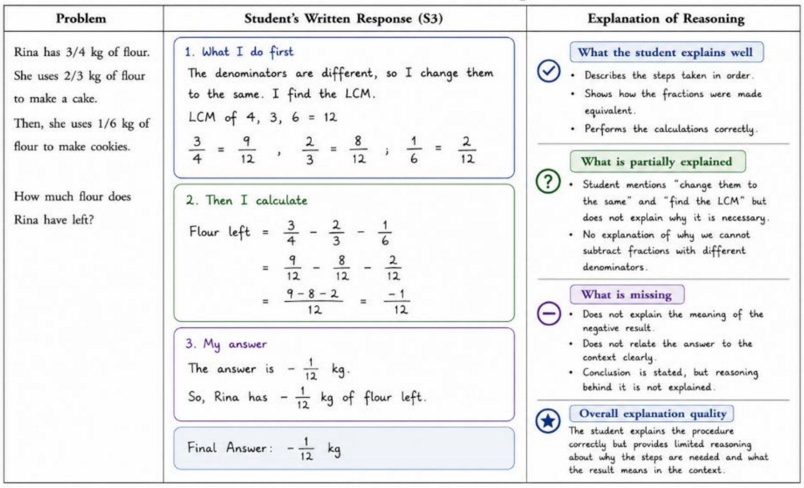

Figure 14. *Medium-Level Student (S3): Partial Explanation of Mathematical Reasoning in Fraction Problem Solving*

Figure 14 shows that the student described procedural steps correctly but provided limited explanation regarding the conceptual basis of the selected strategy.

Interview findings further supported this observation:

Researcher: Why did you use this step?

S3: Because I remembered that fractions with different denominators need another step first, but I do not really know why.

This finding suggests that medium-level students possessed procedural awareness but demonstrated incomplete conceptual understanding during explanatory processes. Low-level students (S5–S6) demonstrated substantial difficulties in articulating reasoning processes. Written responses frequently contained isolated calculations without accompanying explanations or logical connections among solution steps. Figure 15 presents a representative response from S5 illustrating explanatory difficulties.

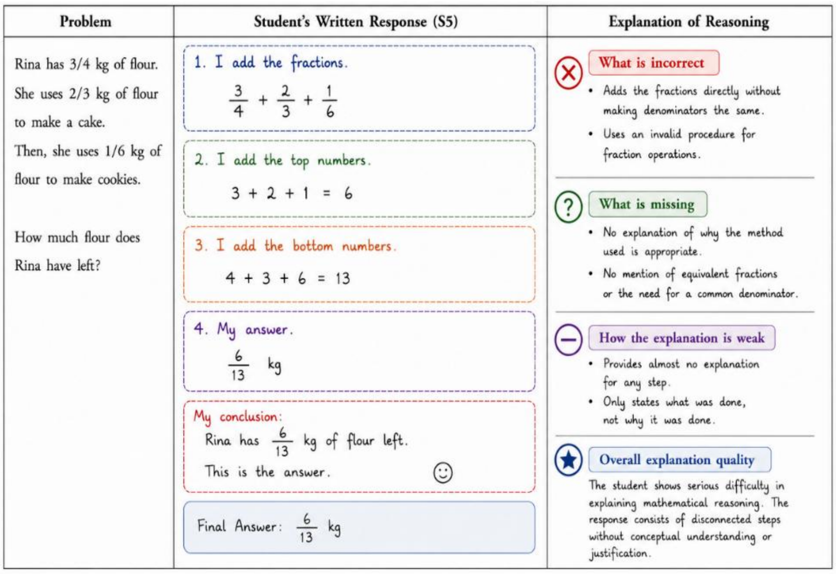

Figure 15. *Low-Level Student (S5): Difficulty in Explaining Mathematical Reasoning in Fraction Problem Solving*

Figure 15 illustrates that the student provided fragmented procedures without explaining the conceptual reasoning underlying the response.

Interview findings further revealed limited explanatory ability:

Researcher: Can you explain why you answered this way?

S5: I do not know how to explain it; I just added the numbers.

This response suggests that low-level students relied primarily on procedural actions without constructing meaningful conceptual explanations. Overall, cross-case comparison indicates that explanatory ability progressively decreased across categories. High-level students demonstrated coherent and conceptually grounded explanations, medium-level students exhibited partial explanatory processes with limited conceptual support, whereas low-level students showed fragmented explanations characterized by weak connections between procedures and conceptual understanding.

Summary of Major Findings

Cross-case synthesis of the findings revealed three major patterns regarding students' mathematical critical thinking in solving fraction problems involving different denominators. First, students' mathematical critical thinking abilities varied substantially across ability categories and critical thinking indicators. High-level students demonstrated relatively complete critical thinking processes characterized by systematic interpretation, appropriate analytical strategies, effective evaluation, logically supported conclusions, and coherent explanations. Medium-level students generally exhibited partial understanding across indicators, demonstrating procedural knowledge accompanied by uncertainty and incomplete conceptual reasoning. In contrast, low-level students demonstrated fragmented critical thinking processes characterized by procedural errors, conceptual misconceptions, and limited reasoning abilities. Second, additive misconceptions emerged as the most frequently observed conceptual difficulty, particularly among students in the low-ability category. Students frequently treated fractions as separate whole numbers and directly added numerators and denominators without considering denominator equivalence. This misconception was consistently reflected in written responses and supported by interview findings, suggesting that students experienced difficulties in recognizing relationships among fractional quantities and contextual meanings. Third, the findings indicate interconnected patterns across critical thinking indicators. Students who demonstrated stronger interpretation abilities generally tended to exhibit more systematic analytical processes, stronger evaluative reasoning, and more coherent explanations. Conversely, students who experienced difficulties during initial interpretation frequently demonstrated weaknesses in subsequent reasoning stages. These findings suggest that mathematical critical thinking processes did not operate independently but rather functioned as interconnected dimensions within students' problem-solving activities. The findings indicate that students' mathematical critical thinking in fraction problem solving was influenced not only by procedural competence but also by students' ability to construct conceptual understanding and connect contextual information with mathematical reasoning processes.

DISCUSSION

Overall, the findings indicate that seventh-grade students' mathematical critical thinking abilities in solving fraction problems involving different denominators were unevenly distributed across critical thinking indicators and ability categories. Cross-case analysis revealed substantial variation in students' abilities to interpret problems, analyze mathematical relationships, evaluate procedures, formulate conclusions, and explain reasoning processes. High-level students demonstrated relatively coherent thinking patterns across indicators, whereas medium- and low-level students exhibited fragmented reasoning characterized by procedural uncertainty and conceptual misunderstanding. These findings support the view that mathematical critical thinking operates through interconnected reasoning dimensions rather than isolated cognitive skills (Ennis, 2015; Sari, 2020; Astuti et al., 2026).

A notable pattern emerging from the findings is the hierarchical organization of critical thinking indicators. Interpretation appeared to function as the most accessible dimension because most students were able to identify information embedded in contextual situations. This pattern may occur because contextual representations reduce the abstract nature of mathematical ideas and provide an initial cognitive bridge between everyday experiences and symbolic representations. Students are often able to recognize familiar situations before engaging in more demanding reasoning processes. Similar studies have demonstrated that contextual and RME-oriented learning environments support students' engagement and mathematical thinking by linking abstract concepts to meaningful experiences (Tambunan & Mahmudi, 2024; Lestari et al., 2023). Therefore, contextual environments appear to facilitate initial stages of reasoning construction, although such support may not automatically extend to more complex cognitive processes.

In contrast, inference emerged as the most difficult indicator, particularly among medium- and low-level students. Students frequently experienced difficulties transforming previous reasoning processes into logically valid conclusions. Errors occurring during interpretation and analysis often propagated into later stages, resulting in unsupported conclusions and inaccurate judgments. This finding suggests that inferential reasoning requires students not only to perform procedures but also to synthesize information, evaluate relationships, and justify conclusions simultaneously. Inferential perspectives in mathematics education similarly emphasize that conclusions emerge through coordinated reasoning processes rather than isolated computational actions (Derry, 2017; Seidouvy & Schindler, 2020). Likewise, Pfannkuch et al. (2014) highlighted that inferential thinking requires integration across multiple cognitive activities. Thus, weaknesses in inference may reflect accumulated reasoning difficulties occurring throughout earlier stages of problem solving rather than deficiencies confined solely to concluding processes.

The explanation indicator also demonstrated considerable weaknesses, particularly regarding students' ability to provide conceptual justification for selected procedures. Medium-level students generally described procedural steps correctly but frequently experienced difficulties explaining why those procedures were appropriate. This finding suggests that students tended to possess procedural awareness without sufficiently developed conceptual understanding. Hurrell (2021) argued that conceptual and procedural knowledge should function in conjunction because effective mathematical understanding depends upon meaningful integration of both dimensions. Similarly, Norqvist (2018) emphasized that explanation processes strengthen mathematical reasoning because students are required to organize and justify relationships among mathematical ideas. Consequently, students' difficulties in explanation may indicate broader limitations in conceptual organization rather than merely communication weaknesses.

A particularly important finding emerging from this study was the dominance of additive misconceptions among low-level students. Students frequently treated fractions as separate whole-number entities and directly combined numerators and denominators without considering denominator equivalence. Such responses appear to reflect deeper conceptual misunderstandings rather than isolated procedural mistakes. Previous studies similarly identified whole-number bias and incorrect magnitude reasoning as major barriers to fraction learning (DeWolf & Vosniadou, 2015; González-Forte et al., 2023). Xu et al. (2024) further suggested that fraction misconceptions emerge through interactions among conceptual understanding, confidence, and reasoning abilities. Moreover, conceptual understanding profiles influence how students represent and interpret mathematical ideas during problem solving (Primadoni et al., 2025). Therefore, additive misconceptions identified in this study appear to indicate broader conceptual difficulties that simultaneously influence multiple dimensions of mathematical critical thinking.

An important contribution of this study lies in demonstrating how weaknesses occurring at early stages of reasoning tend to coincide with difficulties in later critical thinking processes. Unlike previous studies that primarily focused on overall achievement in mathematical critical thinking, the present study illustrates how interpretation, analysis, evaluation, inference, and explanation operate as interconnected dimensions within students' reasoning processes. Students demonstrating stronger interpretation abilities generally exhibited more systematic analytical reasoning and more coherent explanations, whereas students experiencing difficulties in early stages frequently demonstrated fragmented reasoning throughout later processes. This finding contributes to a more detailed understanding of how mathematical critical thinking develops across multiple dimensions during fraction problem solving.

From a pedagogical perspective, the findings suggest that instructional practices should move beyond procedural fluency and emphasize opportunities for conceptual construction and reflective reasoning. Learning activities may incorporate tasks requiring students to compare fraction representations, justify procedural decisions, evaluate alternative strategies, and verbalize mathematical reasoning through guided questioning and contextual situations. Such instructional conditions may facilitate more balanced development of mathematical critical thinking abilities across indicators and support deeper conceptual understanding of fraction concepts. Several limitations should be considered when interpreting these findings. First, the study involved a relatively small number of participants because emphasis was placed on detailed exploration of students' cognitive processes rather than statistical generalization. Second, the investigation focused only on fraction problems involving different denominators within a single educational context. Future studies may extend the investigation to broader mathematical domains and incorporate additional variables such as metacognitive processes, mathematical anxiety, or classroom discourse interactions to obtain a more comprehensive understanding of students' mathematical critical thinking development.

CONCLUSIONS

This study concludes that seventh-grade students demonstrated uneven mathematical critical thinking abilities in solving fraction problems involving different denominators within Realistic Mathematics Education (RME) contexts. Cross-case analysis revealed substantial differences across interpretation, analysis, evaluation, inference, and explanation indicators. High-level students generally demonstrated coherent and systematic reasoning processes, whereas medium- and low-level students exhibited fragmented thinking characterized by procedural uncertainty, incomplete conceptual understanding, and difficulties in articulating mathematical reasoning. A major finding of this study was the dominance of additive misconceptions, particularly among low-level students. Rather than representing isolated procedural errors, these misconceptions appeared to reflect broader conceptual difficulties in understanding fractions as integrated numerical representations. Furthermore, the findings suggest that mathematical critical thinking indicators function as interconnected dimensions, in which difficulties occurring during earlier stages of reasoning frequently coincided with limitations in subsequent processes. Interpretation emerged as the most accessible indicator, whereas inference and explanation represented the most challenging dimensions. This study contributes to the mathematics education literature by providing a multidimensional profile of students' mathematical critical thinking through the integration of written responses and interview data. The findings demonstrate how critical thinking processes develop across interconnected indicators during fraction problem solving and provide a more detailed understanding of students' reasoning beyond achievement outcomes alone. Pedagogically, these findings highlight the importance of instructional practices that extend beyond procedural fluency and emphasize conceptual understanding, reflective reasoning, and opportunities for students to justify mathematical thinking. Future research is recommended to involve broader mathematical topics, more diverse participant groups, and additional cognitive variables to obtain a more comprehensive understanding of students' mathematical critical thinking development.

ADDITIONAL INFORMATION

Section	Description
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