

Structural Alignment in Students' Reasoning on Rational Inequalities: A Comenian Hermeneutic Analysis

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Abstract

This study investigates students' reasoning in rational inequalities through a structural–hermeneutic framework interpreted within Comenian epistemology. Rather than examining procedural accuracy alone, the study explores the degree to which students' reasoning aligns with the intrinsic structural order of rational expressions. 32 students completed 3 rational inequality tasks with increasing relational complexity. Written responses and follow-up interviews were analysed sequentially through structural segmentation, coherence analysis, and epistemic interpretation. Across tasks, three stable configurations emerged: coherent rational alignment, procedural performance without integration, and fragmented structural reasoning. Students demonstrating coherent alignment engaged domain restrictions as structural boundaries, interpreted zeros as relational thresholds, and conducted interval-based sign analysis relationally rather than mechanically. Procedural performance without integration reflected operational competence but limited articulation of structural coherence. Fragmented reasoning revealed a breakdown of relational unity, including the decomposition of rational expressions into independent linear components and the omission of discontinuity. As symbolic and relational complexity increased—particularly in tasks requiring quadratic transformation—structural coherence decreased while fragmentation increased. The findings suggest that difficulty with rational inequalities lies not solely in algebraic manipulation but in sustaining epistemic alignment among the recognition of discontinuities, symbolic transformation, and the interpretation of relational signs. Theoretically, the study reframes rational inequality reasoning as a problem of epistemic formation rather than procedural mastery. Within a Comenian perspective, mathematical understanding emerges when cognition conforms to intrinsic structural order. The results highlight the importance of instructional approaches that foreground relational coherence and structural unity in algebraic reasoning.

INTRODUCTION

Rational inequalities constitute a central topic in secondary algebra because their solution requires students to coordinate algebraic manipulation with the structural constraints of rational expressions. Unlike linear inequalities, rational inequalities involve expressions whose validity depends on domain restrictions and sign changes across intervals. Determining the solution set, therefore, requires more than executing algebraic procedures; it requires understanding how the expression's relational structure determines permissible values. In mathematical logic, restriction

principles specify the conditions under which expressions remain meaningful (Jackson & Stokes, 2023), while studies of algebraic systems show that excluding undefined elements is a structural requirement in partial mathematical structures (Borlido & McLean, 2022). Research on instructional example spaces further indicates that structural boundaries strongly influence how learners conceptualize algebraic relations and functions (Uscanga & Cook, 2024). Despite these structural features, classroom instruction often introduces rational inequalities through procedural routines such as determining critical points, excluding values that make the denominator zero, performing sign analysis, and identifying solution intervals. Although these procedures frequently lead to correct answers, studies consistently show that students may carry out the steps without fully understanding the relationships that justify them (Cahyani et al., 2024; Kertil et al., 2025).

This discrepancy between procedural success and structural understanding reflects a broader issue in mathematics education. In many instructional contexts, students' mathematical achievement is evaluated primarily through procedural accuracy and the efficient manipulation of symbolic expressions. While such practices support algorithmic fluency, they do not necessarily ensure that learners grasp the structural relations that organize mathematical objects. The distinction between instrumental and relational understanding illustrates this limitation, indicating that procedural competence may coexist with fragile conceptual understanding (Herheim, 2023). Empirical work also suggests that performance-oriented instructional approaches can improve measurable outcomes while leaving deeper conceptual integration uneven (Tuazon, 2025). From a broader educational perspective, scholars have argued that an excessive emphasis on measurable attainment risks narrowing the epistemic and democratic purposes of mathematics education (Skovsmose, 2023).

Most research addressing these difficulties interprets students' errors primarily in cognitive or pedagogical terms. Misconceptions are commonly described as gaps in conceptual knowledge or incorrect strategies. While such explanations have generated important insights, they rarely address the epistemological dimension of students' reasoning. Students' errors may also reflect incomplete coordination between cognitive activity and the structural order of mathematical objects. From this perspective, difficulties in rational inequalities may indicate varying degrees of epistemic alignment rather than simply procedural failure.

To explore this dimension, the present study draws on the epistemological ideas of John Amos Comenius. In Comenius's philosophy, knowledge is not merely the accumulation of isolated propositions but part of a broader process of intellectual formation oriented toward understanding the rational order of reality (Meyer, 2016). Recent philosophical scholarship has reconstructed this epistemology as an integrated theory of *Bildung* in which ontology, method, and pedagogy are intrinsically connected (Velilla-Jiménez, 2025). Interpretations of Comenius emphasize the cultivation of rational judgment and wisdom through disciplined intellectual development (Osika, 2018), while the *Didaktik* tradition similarly conceptualizes education as a process that supports reflective judgment and intellectual autonomy (Uljens, 2025).

Although renewed interest in Comenian epistemology has emerged in educational philosophy, its implications for interpreting students' mathematical reasoning remain largely unexplored. Mathematics education research typically analyzes misconceptions through cognitive frameworks, whereas philosophical interpretations of epistemic formation are rarely integrated into empirical studies of algebraic reasoning. At the same time, recent research highlights the importance of mathematical justification for supporting deeper reasoning and equitable participation in mathematical activity (Thanheiser & Sugimoto, 2022). Instructional activities involving comparison, explanation, and generalization have also been shown to strengthen

relational reasoning beyond procedural replication (Öz & Çiftci, 2024). Similarly, design-based approaches to epistemic development emphasize how structured engagement with mathematical relationships can foster conceptual growth (Lehrer et al., 2024).

Broader theoretical perspectives further suggest that reasoning develops through relational structures rather than isolated cognitive processes. Relational sociology conceptualizes thinking as emerging from structured relations among actors and ideas (Rawolle & Lingard, 2022), while philosophical discussions of epistemic injustice suggest that fragmented reasoning may reflect structural conditions that shape intellectual participation (Marabini, 2022). Historical analyses of mathematics curricula also demonstrate how curricular transformations influence intellectual orientations toward mathematical reasoning (Matos & Almeida, 2023). Research on technologically mediated mathematical activity similarly suggests that such environments can support creative and reflective epistemic development (Abramovich & Freiman, 2023). In addition, formative evaluation frameworks emphasize the structuring of reflective judgment as a central component of intellectual development in educational design (de Wijse-van Heeswijk & Kriz, 2023).

Despite these diverse perspectives, an integrative framework that connects relational theory, epistemic justice, historical curriculum analysis, and Comenian Bildung for interpreting students' algebraic reasoning remains largely absent. This study addresses that gap by examining how students justify their reasoning when solving rational inequalities. Rather than focusing solely on procedural correctness, the analysis investigates how students coordinate algebraic relations, domain restrictions, and logical justification in their solutions. By analyzing students' written work and interview explanations through a structural–hermeneutic perspective informed by Comenian epistemology, the study aims to identify patterns of reasoning that reflect different degrees of epistemic coherence in students' understanding of rational inequalities.

METHODS

Philosophical–Hermeneutic Positioning

This study is situated within a philosophical–hermeneutic interpretive paradigm that integrates empirical classroom analysis with a Comenian theory of knowledge. Within this perspective, mathematical understanding is interpreted not simply as a measurable cognitive outcome but as a formative process in which reasoning becomes aligned—more or less coherently—with the structural relations inherent in mathematical objects. Such positioning resonates with hermeneutic accounts that conceptualise mathematical understanding as engagement with structured meaning rather than as the decoding of symbolic procedures alone (Carl, 2022). Accordingly, students' written and verbal solutions are treated as sites of meaning construction rather than as neutral indicators of correctness. Hermeneutic analysis operates through a whole–part interpretive dynamic: individual procedural steps are examined in relation to the coherence of the overall reasoning structure. In the case of rational inequalities, elements such as domain restrictions, interval segmentation, and sign relations gain meaning only through their integration within the expression as a whole. The philosophical framework did not determine analytical categories in advance. Instead, structural patterns of reasoning were first identified empirically and subsequently interpreted in relation to epistemic formation. This analytic sequence preserves interpretive openness while situating the findings within a broader conception of knowledge as rational formation (Vanegas et al., 2023). In this sense, the philosophical orientation serves as an interpretive horizon that guides analysis rather than a prescriptive template imposed on students' reasoning.

Research Design

The study employed a qualitative interpretive case study design to examine how students reason when solving rational inequalities. The objective was not to establish causal relationships but to understand how students coordinate structural elements of rational expressions in their reasoning. In qualitative inquiry, case study designs are particularly appropriate when the researcher seeks to develop an in-depth understanding of a phenomenon within its real-life educational context, especially when the boundaries between the phenomenon and the context are clearly defined (Mulisa, 2022). Qualitative case studies enable researchers to explore processes of meaning-making by examining participants' experiences and reasoning within a bounded system such as a specific classroom, instructional unit, or learning activity. In the present study, the bounded case consisted of a single instructional unit on rational inequalities. This topic provides a clearly defined mathematical structure involving domain restrictions, identification of critical points, sign relations across intervals, and interval reasoning. These structural components provided a coherent analytical framework for examining how students coordinate procedural operations with relational understanding. Prior studies in mathematics education have shown that students often encounter conceptual obstacles in interpreting algebraic structures, particularly when attempting to connect symbolic manipulation to underlying structural meaning (Diana, 2024). Such difficulties underscore the need to investigate students' reasoning processes beyond procedural correctness. Moreover, research on algebraic thinking emphasises that meaningful algebraic understanding involves coordinating symbolic, relational, and structural reasoning when interpreting mathematical expressions (Sibgatullin et al., 2022). Consistent with traditions of educational case study research that aim to capture contextualised learning processes, the design enabled systematic interpretation of students' written and verbal reasoning as manifestations of their underlying epistemic organisation in algebraic thinking.

Research Context and Participants

The study was conducted in a Grade X mathematics classroom at a public senior high school following instruction on rational inequalities. Participants included 32 students who had completed the instructional unit, one mathematics teacher responsible for delivering the instruction, and four expert practitioners—two mathematics education lecturers and two experienced secondary mathematics teachers—who contributed to analytic validation. All students completed the written diagnostic tasks ($n = 32$), and their responses were initially reviewed to preserve the integrity of the classroom case and to enable structural comparison across participants. During the preliminary examination, several responses were found to lack sufficient procedural evidence to permit reliable structural segmentation. These responses typically contained only final answers or incomplete symbolic work without traceable reasoning steps. Such responses were retained for descriptive reference but were excluded from categorical distribution analysis because they did not provide analyzable structural evidence.

Consequently, the frequency distributions reported in the Results section refer only to structurally analyzable responses, while the full set of responses ($n = 32$) remained part of the broader qualitative interpretation of reasoning patterns. For the interview phase, purposive maximum-variation sampling was employed to capture diverse reasoning configurations identified during preliminary coding. Nine students were selected to represent three empirically observed reasoning patterns: coherent structural integration (three students), procedurally correct solutions with limited justification (three students), and structurally fragmented reasoning (three students). The purpose of this sampling strategy was to obtain analytic depth across contrasting reasoning structures rather than to achieve statistical representativeness.

Instructional Context

Instruction followed the regular curriculum and addressed the core structural components of rational inequalities. Students were introduced to domain restrictions in rational expressions, the identification of zeros in both the numerator and the denominator, the construction of sign charts to analyse function behaviour across intervals, and the determination of solution sets through interval reasoning. These elements were presented sequentially to support procedural fluency and introduce the structural logic underlying rational expressions. The study did not involve instructional intervention or experimental manipulation. Instead, the classroom served as a naturally occurring context in which students' engagement with domain boundaries, sign relations, and interval continuity could be examined as expressions of their reasoning processes.

Data Collection

Data were collected from four complementary sources to support triangulated analysis. First, students completed three written diagnostic tasks involving rational inequalities. The tasks were designed with increasing structural complexity and required students to identify domain restrictions, determine critical points, analyse sign relations across intervals, and justify the resulting solution sets. Second, semi-structured interviews were conducted with the selected students to explore their reasoning processes in greater depth. Interview questions focused on students' interpretations of domain restrictions, interval boundaries, and sign interactions within rational expressions. Third, classroom observations were conducted during the instructional period. Field notes documented teacher explanations, representational strategies, and students' engagement with structural features such as sign charts and interval diagrams. Fourth, an expert focus group discussion was conducted with mathematics education lecturers and experienced teachers. Participants reviewed anonymised student responses and preliminary coding categories to refine interpretations and strengthen analytic transparency. The session was recorded and summarised to document analytic decisions.

Analytical Framework

Data analysis proceeded through three sequential stages that combined structural coding with interpretive analysis. The analytical procedure progressed from descriptive identification of reasoning components to interpretive examination of structural coherence. In the first stage, students' written and verbal responses were segmented according to four structural components inherent in rational inequalities: domain awareness, identification of critical points, relational sign reasoning, and interval interpretation. These components were derived directly from the mathematical structure of rational inequalities rather than from philosophical assumptions. Coding at this stage was descriptive and data-driven, focusing on identifying the presence, completeness, and mathematical correctness of each structural element within students' solutions.

In the second stage, the analysis examined how these components were integrated within each solution. The focus shifted from identifying isolated procedural elements to evaluating the coherence of the reasoning structure as a whole. Coherence was assessed using three criteria: explicit justification of procedural transitions, recognition of relational dependencies within the rational expression, and logical continuity between symbolic manipulation and interval conclusions. Cross-case comparison was then conducted across student responses to identify recurring configurations of reasoning and to distinguish stable structural patterns from incidental procedural errors.

In the third stage, the empirically identified reasoning configurations were interpreted in relation to epistemic formation. Based on recurring structural patterns, three interpretive categories emerged: coherent rational alignment, procedural performance without integration, and

fragmented structural reasoning. These categories describe varying degrees of structural coherence in students' reasoning rather than fixed indicators of student ability. Importantly, categorisation was conducted only after structural segmentation and coherence analysis were completed, ensuring that interpretive labels emerged from empirical patterns rather than being imposed a priori.

Validity and Trustworthiness

Several strategies were implemented to ensure methodological rigor. Data triangulation was achieved through written tasks, interviews, classroom observations, and expert discussions. Member checking was conducted with interview participants to verify the accuracy of interpretative summaries. Peer debriefing with mathematics education experts supported refinement of coding categories and analytic interpretations. An audit trail documented coding procedures, category development, and interpretive decisions. Analytic memos were also maintained to track emerging insights during the coding process. Particular attention was given to maintaining balance between inductive coding and theoretical interpretation to prevent philosophical assumptions from being imposed prematurely on the data.

Ethical Considerations

Informed consent was obtained from all students and their legal guardians prior to data collection. Participation was voluntary, and students were informed that their decision to participate or withdraw would not affect their academic standing. All data were anonymised to protect participants' identities. Identifying information was removed from written responses, interview transcripts, and observation records. Institutional approval was obtained from the relevant educational authority before the study commenced.

RESULT

All 32 students submitted written responses to each of the three diagnostic tasks. In accordance with the analytical procedure described in the Methods section, each response was first examined to determine whether sufficient procedural evidence was present to permit structural segmentation. Responses that contained only final answers or incomplete symbolic work without traceable reasoning steps were retained for descriptive reference but were excluded from categorical distribution analysis. Consequently, the frequency distributions reported in the tables below refer only to structurally analyzable responses, while the full set of responses remained part of the broader qualitative interpretation of students' reasoning. Following the analytical framework outlined in the Methods section, the analysis proceeded from structural segmentation to coherence evaluation, then to interpretive categorisation. Students' written solutions were initially examined with respect to four structural components inherent in rational inequalities: domain awareness, identification of critical points, relational sign reasoning, and interval interpretation. The integration of these components was then evaluated through coherence analysis, focusing on the justification of procedural transitions, recognition of relational dependencies, and continuity between symbolic manipulation and interval conclusions. Through cross-case comparison, three recurring configurations of reasoning emerged from the data: coherent rational alignment, procedural performance without integration, and fragmented structural reasoning. These configurations were not imposed a priori but developed inductively through the sequential process of structural coding and coherence analysis before being interpreted epistemically. Accordingly, the results reported in this section describe differences in the structural coherence of students' reasoning rather than differences in achievement or procedural correctness alone.

1. *Structural Patterns in Solving $(x + 5)/(x - 5) < 0$*

The first task required students to engage the rational inequality as a structured mathematical object. Students were expected to recognise domain restrictions, determine critical points arising from zeros in the numerator and denominator, segment the number line into analytically meaningful intervals, conduct systematic sign testing, and justify the resulting solution set. The task therefore demanded not only procedural execution but structural coordination across multiple relational components.

Table 1. Student Response

| Category of Student Response | Number of Students |
|-------------------------------|--------------------|
| Correct with justification | 19 |
| Correct without justification | 4 |
| Incorrect | 5 |

The distribution indicates that the majority of students (19 of 28 structurally analyzable responses) demonstrated coherent rational alignment, characterised by integrated reasoning involving domain awareness, identification of critical points, and relational sign analysis. A smaller group (4 responses) exhibited procedural performance without integration, arriving at correct solutions while offering limited structural justification. Five responses reflected fragmented structural reasoning, in which the rational inequality was interpreted through disconnected symbolic procedures rather than as an integrated relational structure.

Category 1: Correct with Justification

Students in this category exhibited a coherent, structural progression in their reasoning. Their solutions began with explicit acknowledgement that the denominator cannot equal zero, thereby identifying the domain boundary embedded in the rational expression. This recognition functioned as more than a procedural reminder; it signalled awareness of structural discontinuity. Using this boundary condition, students determined the zeros of both the numerator and the denominator and interpreted them as decisive structural thresholds. These points reorganised the number line into intervals within which the sign behaviour of the rational expression could be meaningfully examined. Interval segmentation was followed by systematic sign testing. Importantly, sign testing was not treated as an isolated substitution but as relational verification: students examined how the numerator and denominator interact within each interval. The solution interval was then selected on the basis of sign opposition, consistent with the inequality’s condition of negativity. The reasoning culminated in an explicit justification that linked domain restriction, interval analysis, and relational sign behaviour into a unified explanation.

A representative student response illustrates this structural coherence:

“The denominator cannot be zero. I found $x = -5$ and $x = 5$. I divided the number line at these points and tested values in each region. Since the inequality is less than zero, I looked for intervals where the numerator and denominator have opposite signs.”

This explanation reflects epistemic coordination across multiple structural dimensions. Domain awareness indicates recognition of discontinuity; identification of critical points reflects structural threshold reasoning; interval segmentation mediates between symbolic representation and relational behavior; and the interpretation of negativity through sign opposition reveals understanding of the rational expression as a unified relational system. The student does not

manipulate symbols in isolation but instead articulates the internal order that governs the inequality.

Table 2. Correct with Justification

| Conceptual Aspect | Manifestation in Responses |
|-------------------|---|
| Definition | The denominator cannot be zero, explicitly stated |
| Critical Points | $x = -5$ and $x = 5$ identified correctly |
| Sign Analysis | Systematic interval testing |
| Interpretation | Interval chosen based on sign opposition |
| Justification | Conceptual explanation provided |

The internal consistency across these elements indicates high structural integration. Symbolic manipulation, interval reasoning, and relational interpretation operate in coordination rather than in isolation. Correctness here is not merely computational accuracy but coherence across structural components.

Category 2: Correct without Justification

Students in this category reached the correct solution interval but did not articulate the structural logic underlying their reasoning. Although domain and critical points were often identified, the relational meaning of these steps remained implicit.

For example:

“Because between -5 and 5 I tried -1 and it worked.”

Interview data suggest reliance on single-point verification rather than systematic relational reasoning. Sign testing was sometimes reduced to checking a single representative value, without explicitly articulating why sign opposition governs negativity. Domain awareness was mentioned only occasionally and not conceptually elaborated.

Table 3. Correct without Justification

| Conceptual Aspect | Manifestation in Responses |
|-------------------|-------------------------------|
| Definition | Mentioned but unexplained |
| Critical Points | Identified mechanically |
| Sign Analysis | Based on isolated test values |
| Interpretation | Correct but weakly reasoned |
| Justification | Minimal or absent |

These responses demonstrate procedural fluency but limited explanatory depth. Structural components are present; however, their relational interdependence remains weakly articulated. Local operations are performed correctly, yet global coherence is fragile.

Category 3: Incorrect

Students classified as incorrect exhibited structural fragmentation. A common pattern involved decomposing the rational inequality into two separate linear inequalities.

A typical response stated:

“I solved $x + 5 < 0$ and $x - 5 < 0$ separately.”

Here, the numerator and denominator were treated as independent expressions rather than as components of a relational fraction. Domain restriction was frequently omitted, and interval-based sign reasoning was absent. The inequality was reduced to disconnected symbolic manipulations, indicating failure to interpret the rational expression as an integrated structural system.

Table 4. Incorrect

| Conceptual Aspect | Manifestation in Responses |
|-------------------|------------------------------|
| Definition | No domain awareness |
| Critical Points | Treated as independent roots |
| Sign Analysis | Omitted |
| Interpretation | Based on false decomposition |
| Justification | Absent or flawed |

These responses reveal a disconnection between symbolic components and relational structure. The rational inequality is no longer understood as a system governed by discontinuity and sign interaction, but is reduced to fragmented linear procedures. Structural misinterpretation replaces relational reasoning. Overall, the findings from this task indicate that differentiation among students lies not solely in solution accuracy, but in the degree to which symbolic manipulation, domain awareness, interval segmentation, and relational sign interpretation are integrated into a coherent epistemic structure.

2. Structural Patterns in Solving $(x + 2)/(x - 5) \leq 2$

The second task introduced an additional structural demand: algebraically transforming the inequality into a single rational expression before sign analysis. Unlike the first task, students were required to reorganise the expression symbolically before relational evaluation could occur. This increased the structural complexity of the reasoning process, as transformation, domain awareness, and relational sign interpretation had to operate in coordination.

Table 5. Category of Student Response

| Category | Number |
|-------------------------------|--------|
| Correct with justification | 15 |
| Correct without justification | 5 |
| Incorrect | 8 |

Compared to the previous task, the number of fully justified responses decreased, while incorrect and partially articulated responses increased. This shift suggests that the added demand of structural transformation introduced additional epistemic strain.

Category 1: Correct with Justification

Students in this category demonstrated integrated structural reasoning across symbolic transformation and relational analysis. Their work began by rewriting the inequality into standard rational form:

$$\frac{x + 2}{x - 5} \leq 2 \Rightarrow \frac{x + 2 - 2(x - 5)}{x - 5} \leq 0 \Rightarrow \frac{-x + 12}{x - 5} \leq 0$$

This transformation was not performed mechanically; rather, it functioned as a structural reorganisation that made relational sign behaviour accessible to systematic analysis. Following the transformation, students explicitly identified $x = 5$ as a domain exclusion and $x = 12$ as the zero of the numerator. These values were interpreted as structural thresholds that partitioned the number line into intervals over which sign relations could be examined.

Students then conducted systematic sign testing across all intervals and interpreted the inequality condition with careful attention to boundary inclusion. In several cases, students explicitly justified why $x = 5$ must be excluded due to discontinuity, while $x = 12$ was included because the inequality was non-strict.

A representative explanation stated:

“I moved everything to one side to make it equal to zero. Then I simplified to do sign testing. I found that $x = 5$ is undefined, so I excluded it. I tested values in each interval to see where the expression is less than or equal to zero.”

This reasoning reflects structural integration at multiple levels. The student recognizes that transformation into standard form is necessary to expose relational sign structure. Domain exclusion is treated as a structural constraint rather than an afterthought. Interval segmentation mediates between symbolic manipulation and relational evaluation, and boundary inclusion is interpreted in light of the inequality’s semantic condition.

Table 6. Correct with Justification

| Conceptual Aspect | Manifestation |
|-------------------|---|
| Definition | $x \neq 5$ explicitly excluded |
| Equation Setup | Correct transformation into standard form |
| Critical Points | $x = 5$ and $x = 12$ identified |
| Sign Testing | Systematic across all intervals |
| Justification | Full relational explanation |

The consistency across these elements indicates high epistemic coherence. Symbolic transformation, awareness of discontinuity, interval reasoning, and interpretation of inequality operate as mutually reinforcing components of a unified structural understanding.

Category 2: Correct without Justification

Students in this group reached correct solution intervals but did not articulate the structural logic underlying their reasoning. Transformation steps were sometimes implied but not fully shown. Boundary inclusion or exclusion was rarely justified explicitly.

For example:

“I followed what we usually do with sign charts.”

Interview data revealed a reliance on procedural templates rather than on relational articulation. Students often performed transformation and sign testing but did not explain why these steps were necessary or how discontinuity affects the solution set. In some cases, only one test value per interval was checked without systematic verification.

Table 7. Correct without Justification

| Conceptual Aspect | Manifestation |
|-------------------|-----------------------------|
| Definition | Partial or implicit mention |

| | |
|-----------------|------------------------------------|
| Equation Setup | Sometimes implied, not fully shown |
| Critical Points | Mechanically identified |
| Sign Testing | Inconsistent or weakly explained |
| Justification | Minimal or absent |

These responses demonstrate operational competence but limited structural transparency. The procedural sequence is executed, yet the relational meaning of transformation and boundary logic remains under-articulated. Structural elements are present but not fully integrated into a coherent explanatory framework.

Category 3: Incorrect

Incorrect responses revealed pronounced structural fragmentation, particularly during the algebraic transformation stage. Many students failed to convert the inequality into standard rational form. Instead, they treated the inequality as two independent linear inequalities.

A typical response stated:

“I solved $x + 2 \leq 2$ and $x - 5 \leq 2$ separately.”

Here, the rational structure was decomposed into unrelated linear comparisons, eliminating the relational interaction between the numerator and the denominator. Domain exclusion was frequently misunderstood or omitted, and some students included $x = 5$ in their final solution, disregarding discontinuity.

Table 8. Incorrect

| Conceptual Aspect | Manifestation |
|-------------------|---|
| Definition | Misunderstood or ignored |
| Equation Setup | Treated as separate linear inequalities |
| Critical Points | Miscomputed or misapplied |
| Sign Testing | Omitted or incorrect |
| Justification | Procedurally flawed |

These responses indicate a breakdown at the level of structural reorganization. The inability to transform the inequality into a single rational expression prevents the analysis of relational signs. The rational inequality is reduced to fragmented symbolic operations lacking global coherence. Overall, this task reveals that the introduction of algebraic transformation intensifies structural demands. While some students successfully integrate transformation with relational reasoning, others exhibit partial articulation or structural collapse. The increase in incorrect and weakly justified responses suggests that symbolic reorganization constitutes a critical threshold for epistemic coherence in rational inequality reasoning.

3. Structural Patterns in Solving $(x + 3)/(x - 1) \geq x$

The third task imposed the highest structural demand among the three problems. Unlike the previous items, students were required not only to recognize discontinuity and conduct sign analysis, but also to perform quadratic transformation after relocating all terms to one side of the inequality. This task, therefore, required full structural integration: symbolic reorganization, domain awareness, determination of higher-order critical points, interval partitioning, and interpretation of relational signs had to operate cohesively.

Table 9. Category of Student Response

| Category | Number |
|-------------------------------|--------|
| Correct with justification | 12 |
| Correct without justification | 7 |
| Incorrect | 9 |

The distribution reveals a marked shift compared to previous tasks. Fully justified structural reasoning decreased further, while incorrect responses increased substantially. The majority of students either relied on partial procedural execution or exhibited structural fragmentation. This pattern indicates heightened epistemic strain under increased relational and symbolic complexity.

Category 1: Correct with Justification

Students in this category successfully reorganized the inequality into a unified rational expression:

$$\frac{x+3}{x-1} - x \geq 0 \Rightarrow \frac{x+3-x(x-1)}{x-1} \geq 0 \Rightarrow \frac{-x^2+2x+3}{x-1} \geq 0$$

This transformation was essential for exposing the relational sign structure of the inequality. Students explicitly identified the domain restriction $x \neq 1$, recognizing discontinuity as a structural constraint. They then solved the quadratic equation to determine its zeros, which, together with the point of discontinuity, partitioned the number line into analytically significant intervals. Sign testing was conducted systematically across all intervals. Students interpreted non-negativity relationally, evaluating how the quadratic numerator and linear denominator interact within each segment. Boundary inclusion and exclusion were determined with reference to both inequality type and domain constraints.

A representative explanation stated:

“I moved everything to one side and simplified. The expression is undefined at $x = 1$, so I excluded it. Then I found where the numerator equals zero and tested intervals to see where the expression is greater than or equal to zero.”

This reasoning reflects structural coherence across multiple dimensions. Symbolic transformation is treated as a means of revealing internal relational order. Discontinuity is recognized as an inherent feature of the rational structure. Interval segmentation mediates between algebraic form and qualitative behavior. The final justification integrates symbolic manipulation, relational sign interpretation, and domain logic into a unified explanatory structure.

Table 10. Correct with Justification

| Conceptual Aspect | Manifestation |
|-------------------|--|
| Definition | $x \neq 1$ explicitly excluded |
| Equation Setup | Accurate quadratic transformation |
| Critical Points | Correctly derived from numerator and denominator |
| Sign Testing | Systematic across all intervals |
| Justification | Fully aligned with relational and domain logic |

These responses demonstrate full structural integration. Local algebraic procedures and global relational interpretation operate coherently. Correctness here emerges from structural alignment rather than isolated computational accuracy.

Category 2: Correct without Justification

Students in this group executed many symbolic steps correctly but did not fully articulate or verify relational coherence. While transformation and identification of critical points were often present, interval testing was incomplete or insufficiently justified.

For example: *“I thought testing one value per region was enough.”*

Interview data indicate reliance on minimal substitution rather than comprehensive relational verification. Although students recognized domain restrictions and critical points, they did not consistently justify why sign behavior remains stable across intervals. Justification was frequently implicit or omitted.

Table 11. Correct without Justification

| Conceptual Aspect | Manifestation |
|-------------------|----------------------------------|
| Definition | Recognized but implicit |
| Equation Setup | Adequate transformation |
| Critical Points | Identified correctly |
| Sign Testing | Incomplete or minimally verified |
| Justification | Weak or absent |

These responses reveal procedural dominance without full relational integration. Structural elements are present, yet their interdependence is insufficiently articulated. The reasoning appears operationally competent but epistemically fragile.

Category 3: Incorrect

Incorrect responses in this task reveal pronounced structural disintegration. Many students reduced the inequality to scalar statements or detached symbolic manipulations.

A representative example stated:

“Since $3 \geq 0$ is true, the inequality is always true.”

In such cases, students simplified the transformed expression incorrectly and interpreted the resulting scalar inequality as globally valid. Domain restriction was frequently ignored, and sign testing was omitted entirely. The rational inequality was no longer treated as a relational system but collapsed into isolated symbolic statements. Other students misidentified critical points or failed to solve the quadratic equation in the numerator correctly, preventing meaningful interval partitioning.

Table 12. Incorrect

| Conceptual Aspect | Manifestation |
|-------------------|--------------------------------------|
| Definition | Ignored or misinterpreted |
| Equation Setup | Reduced to scalar comparison |
| Critical Points | Misidentified or incorrectly derived |
| Sign Testing | Not performed |
| Justification | Invalid or absent |

These responses reflect a breakdown at the structural level. Symbolic manipulation is disconnected from relational meaning, and discontinuity is not integrated into reasoning. The rational inequality is treated as an arbitrary algebraic expression rather than as an internally ordered system governed by relational sign behavior.

Cross-Task Structural Trend

Across the three tasks, a clear structural progression emerges. As relational and symbolic complexity increased, the proportion of fully justified responses decreased ($19 \rightarrow 15 \rightarrow 12$), while structurally fragmented reasoning increased ($5 \rightarrow 8 \rightarrow 9$). Procedural performance without integration also became more visible ($4 \rightarrow 5 \rightarrow 7$), suggesting that increasing symbolic transformation demands place greater strain on students' ability to sustain coherent structural reasoning. When transformation becomes quadratic and relational dependencies multiply, coherence becomes more fragile. Students who successfully coordinated domain awareness, transformation, interval segmentation, and relational sign interpretation in earlier tasks often struggled to maintain that integration under increased structural demands. Taken together, these findings indicate that differentiation among students lies not merely in procedural accuracy but in the degree of epistemic coherence achieved when symbolic manipulation, discontinuity, and relational reasoning must operate simultaneously within a unified structural framework. The structural configurations reported across the three tasks correspond directly to the analytical progression described in Figure 1. Structural segmentation enabled the identification of domain awareness, critical-point reasoning, and sign relations; coherence analysis distinguished integrated from partially articulated reasoning; and only thereafter were configurations interpreted in terms of epistemic alignment. The findings, therefore, reflect not categorical imposition but the sequential movement from structural coding to coherence evaluation and interpretative synthesis.

DISCUSSION

The findings of this study can be interpreted through a Comenian epistemological lens, in which learning is understood as alignment between human cognition and the intrinsic order (*ordo*) of reality. In early modern Comenian metaphysics, knowledge was conceived not as an accumulation of discrete propositions but as conformity of thought to structured order (Popkin, 1986). From this perspective, rational inequalities are not merely symbolic exercises but structured mathematical objects whose intelligibility depends on internal relational coherence. The three empirically derived configurations—coherent rational alignment, procedural performance without integration, and fragmented structural reasoning—may therefore be interpreted as differing degrees of conformity between student reasoning and the structural order inherent in the rational expression.

In Comenian epistemology, understanding arises when reasoning mirrors the internal structure of the object under inquiry (Nakládálová, 2016). Rational inequalities possess the following structure: they are governed by discontinuities (denominator restrictions), relational thresholds (zeros of the numerator and denominator), interval partitioning, and sign interactions. These elements form an internally ordered whole rather than independent procedures. Students demonstrating coherent, rational alignment engaged the inequality in a manner consistent with this structural unity. Domain restriction was interpreted as recognition of an ontological boundary; critical points functioned as relational thresholds reorganizing sign behavior; interval segmentation mediated between symbolic transformation and qualitative interpretation; and sign testing was relational rather than mechanical. Such reasoning aligns with accounts of mathematical understanding that emphasize structural coherence over procedural decoding (Schaathun, 2022;

Radford, 2015). In Comenian terms, these students exhibited epistemic conformity to *ordo*: their reasoning unfolded in accordance with an internal relational structure rather than by imposing an arbitrary technique.

The configuration labelled procedural performance without integration reflects partial alignment. Students executed appropriate symbolic transformations but did not fully articulate the coherence binding domain restriction, interval structure, and sign interaction. This condition resembles what hermeneutic accounts describe as incomplete integration of parts into a meaningful whole (Jahnke, 2014). From a Comenian standpoint, such reasoning approximates structural order yet lacks synoptic unity. Operational correctness persists, but relational meaning remains under-articulated. The progression across tasks supports this interpretation. As transformation complexity increased—particularly with quadratic reorganization—procedural alignment without structural integration proved unstable. When symbolic restructuring and relational coordination had to occur simultaneously, reasoning struggled to sustain coherence.

Fragmented structural reasoning reflects epistemic misalignment. Students who decomposed rational inequalities into independent linear comparisons or reduced expressions to scalar statements disengaged from the unity of the relational system. Discontinuity was neglected, sign interaction omitted, and transformation detached from interval meaning. In Comenian thought, this represents a breakdown of *syncretism*—the apprehension of relations among parts within a coherent whole (Nakládalová, 2016). Rather than perceiving the numerator and denominator as interdependent within a unified structure, students treated them as separate fragments. The marked rise in fragmentation in the third task suggests that increased relational density intensifies demands for structural integration, a pattern consistent with broader discussions of epistemic alignment in complex reasoning contexts (Kobayashi Hillman et al., 2018).

Across the three tasks, a progressive trajectory emerged: as symbolic and relational complexity increased, coherent alignment decreased while fragmentation intensified. This trajectory supports the interpretation that epistemic formation (*Bildung*) is not the accumulation of procedures but the progressive capacity to coordinate multiple structural relations simultaneously. The structural density of the mathematical object increased from sign opposition to symbolic reorganisation and finally to quadratic transformation with multi-threshold interval reasoning. Students who maintained alignment demonstrated formation consistent with Comenian ideals of unity of knowledge, in which understanding reflects ordered integration rather than fragmented manipulation (Alam Choudhury, 1999).

From a Comenian perspective, instruction should therefore foreground structural order rather than procedural rehearsal alone. Domain restrictions should be framed as ontological boundaries, critical points as relational thresholds, interval partitioning as mediation of relational change, and sign testing as analysis of interaction. Such an orientation aligns with research in mathematics education that emphasises structural understanding in algebraic reasoning (Anggoro & Prabawanto, 2019; Radford, 2015). The analytical progression from structural segmentation to coherence analysis and interpretative synthesis mirrors the hermeneutic movement from parts to whole (Jahnke, 2014). The categories reported in the Results do not impose philosophical abstraction upon the data; rather, they reveal empirically stabilised degrees of conformity between reasoning and structural order. Philosophical interpretation functions as a horizon for situating those patterns within a broader theory of knowledge.

In sum, rational inequality reasoning appears less as a question of procedural competence than of epistemic alignment. Students who conform their reasoning to an internal relational order achieve coherent integration; those who fragment the structure operate without a synoptic

apprehension of unity. In Comenian terms, mathematical understanding is realised when cognition mirrors *ordo*. Rational inequalities thus provide a diagnostic site for observing participation in structured order rather than mere execution of symbolic routine.

CONCLUSIONS

This study examined students' reasoning in rational inequalities through a structural–hermeneutic framework interpreted within Comenian epistemology. The findings indicate that differences in performance are not solely attributable to procedural accuracy but reflect varying degrees of epistemic alignment between students' reasoning and the intrinsic structural order of rational expressions. Across three tasks of increasing relational complexity, three reasoning configurations were identified: coherent rational alignment, procedural performance without integration, and fragmented structural reasoning. Students demonstrating coherent alignment approached rational inequalities as internally ordered systems. Domain restrictions functioned as structural boundaries, zeros as relational thresholds, interval partitioning as meaningful mediation, and sign analysis as relational interaction rather than mechanical substitution. In contrast, students in the procedural configuration carried out appropriate symbolic steps but did not consistently articulate the structural relationships linking them, resulting in reasoning that became unstable as symbolic complexity increased. Fragmented reasoning, most evident in the quadratic task, reflected a breakdown of structural coordination: discontinuities were neglected, expressions were reduced to linear fragments, and inequalities were interpreted as isolated scalar statements detached from interval structure. A clear pattern emerged: as relational density increased—particularly in quadratic transformations—coherent reasoning declined while fragmented reasoning expanded. This suggests that the primary difficulty in rational inequalities lies not merely in algebraic manipulation but in maintaining coordinated integration among domain restriction, transformation, interval mediation, and sign interpretation under greater structural demands. Theoretically, the study positions reasoning in rational inequalities as a process of epistemic formation rather than procedural mastery. Within a Comenian perspective, understanding occurs when cognition aligns with the structured order inherent in the mathematical object. Pedagogically, instruction should therefore emphasize structural unity by treating domain restrictions as boundaries, critical points as relational thresholds, interval segmentation as mediation of change, and sign testing as analysis of interaction. Although the study was conducted in a single context with a limited sample, the findings suggest that reasoning in rational inequalities reflects varying degrees of epistemic alignment with mathematical structure.

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