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Mapping Cognitive Breakdown in Mixed Integer Operations: A Newman Error Analysis of Secondary Students' Problem-Solving

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Abstract

This study aims to analyze Grade VII students' errors when solving mixed-integer operation problems using the Newman Error Analysis (NEA) framework. A qualitative case study design was employed to obtain an in-depth understanding of students' cognitive processes underlying their errors. The participants were 23 students from a junior secondary school in West Java, Indonesia, selected through purposive sampling. Data were collected through a diagnostic test, semi-structured interviews, classroom observations, and document analysis. Student responses were classified into five stages of NEA: reading error, comprehension error, transformation error, process skills error, and encoding error. Data were analyzed using an interactive qualitative model involving data reduction, data display, and conclusion drawing. The findings reveal that students' correct response rates remained below 15% across all items, indicating substantial conceptual and procedural weaknesses. Reading errors were minimal; however, comprehension, transformation, process skills, and encoding errors occurred at consistently high frequencies. The dominant difficulties were related to misunderstandings of the operational hierarchy, particularly the equal precedence of multiplication and division and the left-to-right processing rules. Cross-case analysis showed that similar incorrect answers originated from different cognitive sources, including conceptual rigidity, weak procedural fluency, limited metacognitive monitoring, and affective factors such as confusion and anxiety. This study shows that students' errors in mixed-integer operations follow identifiable cognitive patterns rather than occur randomly. The findings underscore the importance of strengthening conceptual understanding and integrating metacognitive strategies into mathematics instruction to prevent cascading errors in problem-solving.

INTRODUCTION

Mathematics is a foundational discipline in formal education because it systematically cultivates logical, analytical, critical, and structured modes of thinking essential to academic and scientific advancement (Miagusttin et al., 2025). Beyond being a collection of numerical procedures, mathematics represents an abstract system of reasoning that underpins scientific inquiry and epistemic development (Restuti et al., 2025). The development of mathematical proficiency requires conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition, all of which interact in meaningful learning processes (Kilpatrick et al., 2001). Contemporary scholarship further emphasises that fostering higher-order thinking skills in mathematics classrooms demands intentional pedagogical design and structured cognitive engagement. In the era of artificial intelligence, mathematics education is increasingly shaped by digital systems that support reasoning and problem-solving, reinforcing the need for deeper conceptual mastery rather than superficial procedural performance (Van Vaerenbergh & Pérez-Suay, 2022). Although emerging technologies such as generative AI show promise for mathematical reasoning tasks, empirical evidence indicates that algorithmic performance does not replace students' conceptual understanding and error awareness (Dao & Le, 2023). Consequently, strengthening foundational mathematical concepts remains a central priority in secondary education.

One essential topic introduced in Grade VII is mixed-integer operations, which involve integrating addition, subtraction, multiplication, and division into a single problem. Mastery of this topic is critical because it provides the conceptual basis for algebraic reasoning and equation-solving in subsequent grades. However, empirical studies consistently report persistent student difficulties with integer arithmetic, particularly in applying the order of operations, interpreting positive and negative signs, and performing calculations accurately (Miftahudin & Putra, 2025). Misconceptions about associativity and procedural hierarchy often lead to systematic computational errors (Eaves et al., 2025). Research examining learners' mastery of integer operations further identifies conceptual misunderstandings regarding signed numbers as a major obstacle to coherent reasoning (Harun et al., 2023). Descriptive analyses comparing students with and without mathematics difficulties reveal distinct error patterns in integer operations, indicating that weaknesses often stem from fragile conceptual networks rather than mere carelessness (Lin & Riccomini, 2026). These findings align with the view that mathematics, as a deductive discipline, requires a structured conceptual progression to ensure the logical and systematic construction of solutions (Ruseffendi, 2014). Therefore, analysing students' errors in mixed integer operations is essential for diagnosing underlying cognitive gaps.

Student errors in mathematics are not random occurrences but diagnostic indicators of conceptual and procedural breakdowns. Systematic error analysis enables educators to identify specific misconceptions and design targeted interventions (Haryadi & Andriati, 2019). By examining errors at each stage of problem-solving, teachers can anticipate recurring misconceptions and reduce their recurrence (Mulyani & Haerudin, 2021). Broader educational research highlights that identifying and addressing mathematical errors requires both cognitive sensitivity and motivational awareness within classroom practice (Hoth et al., 2022). Cognitive diagnostic modelling has further demonstrated the importance of stage-based analysis in understanding students' latent skill profiles (Paulsen & Valdivia, 2022). Empirical case studies confirm that structured error analysis techniques significantly improve mathematics assimilation among struggling learners (Olaseni & Saziwa, 2024), while large-scale investigations reveal consistent error patterns across diverse secondary school contexts (Nwoke et al., 2024). A recent systematic review emphasises that contemporary research increasingly focuses on mapping error structures to inform instructional design better (Lin et al., 2025). Collectively, these studies affirm that error analysis is a rigorous methodological approach to understanding students' mathematical thinking.

Among established analytical frameworks, Newman Error Analysis (NEA) provides a structured model for identifying student errors according to sequential cognitive stages. NEA categorises errors into reading, comprehension, transformation, process skills, and encoding stages, thereby linking observable mistakes to underlying reasoning processes (White, 2010). The approach has been widely implemented in junior and senior secondary mathematics contexts to diagnose problem-solving difficulties (Ahzan et al., 2022). Empirical investigations that apply

Newman's framework demonstrate its effectiveness in identifying stage-specific breakdowns in statistical reasoning and algebraic problem-solving (Prameshti et al., 2024; Rachmawati et al., 2023). Additional case studies confirm the applicability of NEA in analysing mathematical literacy tasks and classroom-based problem solving (Kusmayadi et al., 2022; Ekasari & Putra, 2024). Recent research further extends Newman's procedure to applied mathematics contexts, emphasising its relevance for systematic diagnostic evaluation (Irianti et al., 2024). A scoping review of international scholarship underscores that structured error frameworks such as NEA remain central to contemporary mathematics education research (Shimizu & Kang, 2025). Moreover, cognitive mapping studies reveal that misconceptions often emerge from interconnected reasoning errors rather than isolated procedural slips (Fitria & Susanto, 2023), frequently triggered by cognitive conflict during concept construction (HR et al., 2023). These findings position NEA as a robust analytical lens for investigating error processes.

Despite the extensive application of Newman's framework, existing studies predominantly emphasise the general identification of error types rather than a comprehensive, stage-by-stage mapping of students' reasoning processes in mixed integer operations at the Grade VII level. While prior research has documented misconceptions in algebraic expressions and symbolic manipulation (Stemele & Asvat, 2024), limited attention has been directed toward systematically analysing how each Newman stage contributes to error propagation in foundational integer arithmetic. This gap indicates the need for a detailed cognitive mapping of errors, specifically in mixed-integer operations, where procedural hierarchy and sign interpretation intersect. Therefore, the present study aims to analyse Grade VII students' errors in solving mixed-integer operations problems using Newman's theoretical framework, to identify the types of errors occurring at each cognitive stage, and to examine the underlying causes of these errors. By providing a comprehensive stage-based analysis, this study seeks to contribute theoretically to the refinement of mathematical error analysis and practically to the development of targeted instructional strategies that minimise recurring misconceptions in secondary mathematics classrooms.

METHODS

This study employed a qualitative case study design. Qualitative inquiry is particularly suitable in mathematics education research when the objective is to explore how learners construct meaning, interpret mathematical representations, and develop reasoning processes within authentic classroom contexts (Na, 2023). A case study design allows researchers to investigate a bounded system intensively and holistically, capturing the complexity of classroom phenomena as they naturally occur (Hofman, 2023). In the present study, the bounded system consisted of Grade VII students' errors when solving mixed-integer operation problems. Rather than focusing solely on students' final answers, the study sought to uncover the philosophical and cognitive dimensions underlying mathematical reasoning and error formation, acknowledging that understanding mathematical thinking requires attention to conceptual interpretation and epistemological positioning (Boadu & Bonyah, 2024). Through this design, the research aimed to provide a contextualised and process-oriented understanding of error patterns in secondary mathematics learning.

The study was conducted in May 2025 at a private Islamic junior secondary school (MTs) in Karawang Regency, West Java, Indonesia. The participants comprised 23 students from Class VII A. They were selected through purposive sampling to align with the research objectives. Purposive or criterion-based sampling is appropriate in qualitative case studies when participants are intentionally selected based on their relevance to the phenomenon under investigation (Douglas, 2022). The inclusion criteria were: (1) students who completed the diagnostic test on mixed integer operations, (2) students who demonstrated identifiable conceptual, procedural, or computational errors, and (3) representation of diverse ability levels (high, medium, and low) to ensure variation in error profiles. The object of analysis was the types and underlying causes of students' errors when solving mixed-integer operation problems.

Data were collected through diagnostic testing, semi-structured interviews, classroom observations, and document analysis to ensure methodological triangulation and strengthen the credibility of findings. The diagnostic test played a central role in identifying students' misconceptions and conceptual weaknesses in learning mixed integer operations, as diagnostic assessment is widely recognized as an effective approach for detecting specific error patterns in mathematics learning (Galeos et al., 2024). The instrument was specifically designed to align with the Newman Error Analysis (NEA) framework, enabling systematic classification of students' errors across the five established stages: reading, comprehension, transformation, process skills, and encoding. Prior to its implementation, the instrument underwent expert validation to ensure content validity, conceptual alignment with mixed integer operations, and consistency with the NEA error taxonomy. Subsequently, a limited pilot test was conducted to examine item clarity, linguistic readability, and the appropriateness of the difficulty level. This refinement process ensured that the instrument was both psychometrically sound and pedagogically appropriate, thereby enhancing the reliability and interpretability of the collected data.

Semi-structured interviews were conducted to explore students' reasoning processes in greater depth. This format allows flexibility while maintaining alignment with predefined analytical constructs, making it suitable for capturing nuanced mathematical thinking (Flückiger & Rathgeb-Schnierer, 2023). Observational notes and students' written work were collected to support contextual interpretation and strengthen analytical triangulation. Document analysis further enabled systematic examination of written responses to identify recurring patterns of misconception and procedural breakdown. Profiling mathematical skills and misconceptions through structured documentation enhances the validity of error classification (Ekasari et al., 2025).

Data analysis followed an interactive qualitative model involving data reduction, data display, and conclusion drawing. During data reduction, written responses, interview transcripts, and observational notes were coded according to the five stages of Newman Error Analysis: reading error, comprehension error, transformation error, process skills error, and encoding error (White, 2010). Systematic classification of error patterns enables structured interpretation of cognitive breakdowns, a strategy commonly applied in error-based research across disciplinary contexts (Neuwinger & Riehle, 2025). Data were organised in tabular and narrative forms to facilitate interpretive synthesis. Conclusion drawing involved identifying dominant error types, tracing their conceptual sources, and examining interrelationships among stages. Interview data were used to triangulate interpretations derived from written responses, strengthening analytical coherence.

To ensure credibility and trustworthiness, methodological triangulation was implemented by comparing findings from diagnostic tests, interviews, and observations. Member checking was conducted by confirming interpretive summaries with participating students to ensure alignment between reported findings and their actual reasoning processes. An audit trail was maintained throughout the study, documenting data collection procedures, coding decisions, analytical memos, and interpretive steps to enhance transparency and dependability. Ethical considerations were carefully observed. Institutional permission was obtained prior to data collection, participation was voluntary, and informed consent was secured from students and guardians.

Participants' identities were anonymised in all documentation and reporting to maintain confidentiality and ethical integrity.

RESULT

Data analysis was conducted on students' written responses to identify the types of errors that emerged in solving mixed integer operation problems. The primary objective of this analysis was twofold: (1) to classify the errors made on each test item and (2) to determine the frequency and percentage of students who answered correctly and incorrectly. The results were first presented quantitatively as the number and percentage of correct and incorrect responses for each item, as shown in Table 1.

Table 1. Percentage of Students' Responses

No.	Problem	Correct (N=23)	Incorrect (N=23)	Percentage Correct	Percentage Incorrect
1	$3 \times (-5) - 4 + (-5)$	0	23	0%	100%
2	$-2 - (-4) + (-5) : (-5)$	0	23	0%	100%
3	$-8 : (-4) + (-2) \times (-3)$	1	22	4.35%	95.65%
4	$2 \times 7 - 6 : 2$	2	21	8.69%	91.30%
5	$5 + 8 : 4 \times 3 - 4$	3	20	13.04%	86.96%

Based on Table 1, the overall level of students' understanding of mixed integer operations was very low. All students (100%) answered Problems 1 and 2 incorrectly. Although a slight improvement was observed in Problems 3, 4, and 5, the percentage of correct responses remained below 15%. Specifically, only one student (4.35%) answered Problem 3 correctly, two students (8.69%) answered Problem 4 correctly, and three students (13.04%) answered Problem 5 correctly. These findings indicate substantial conceptual and procedural difficulties across all test items.

Classification of Errors Based on Newman Error Analysis (NEA)

To obtain a deeper understanding of students' error patterns, each incorrect response was classified using the Newman Error Analysis (NEA) framework, which conceptualizes problem solving as a sequential cognitive process consisting of five stages: Reading Error (RE), Comprehension Error (CE), Transformation Error (TE), Process Skills Error (PSE), and Encoding Error (EE). Reading errors were identified when students miscopied numbers or symbols from the problem statement. Comprehension errors occurred when students failed to understand the meaning of negative signs, parentheses, or the hierarchy of operations. Transformation errors were observed when students selected incorrect operational sequences despite understanding the problem. Process skills errors referred to inaccuracies in performing calculations, particularly in integer operations. Encoding errors were identified when students wrote incorrect or illogical final answers that did not correspond to proper solution procedures. Each student's solution was analyzed step by step, and errors were mapped to the corresponding NEA stage. The frequency and percentage of each error type were then calculated to determine dominant patterns. The distribution of these error categories is presented in Table 2.

Table 2. Percentage of Students' Error Types

No.	Problem	RE	CE	TE	PSE	EE	RE (%)	CE (%)	TE (%)	PSE (%)	EE (%)
1	$3 \times (-5) - 4 + (-5)$	1	23	23	23	23	4.35%	100%	100%	100%	100%
2	$-2 - (-4) + (-5) : (-5)$	1	22	22	23	23	4.35%	95.65%	95.65%	100%	100%
3	$-8 : (-4) + (-2) \times (-3)$	2	22	22	22	22	8.69%	95.65%	95.65%	95.65%	95.65%
4	$2 \times 7 - 6 : 2$	3	21	21	21	21	13.04%	91.30%	91.30%	91.30%	91.30%
5	$5 + 8 : 4 \times 3 - 4$	0	20	20	20	20	0%	86.96%	86.96%	86.96%	86.96%

The data indicate that the dominant errors occurred in four categories: comprehension (CE), transformation (TE), process skills (PSE), and encoding (EE), with percentages consistently exceeding 86% across items and reaching 100% in several cases. In contrast, reading errors (RE) were minimal, ranging from 0% to 13.04%. This suggests that most students correctly read and recognized the problem statements; however, substantial difficulties arose after the reading stage, particularly in understanding and applying the operational rules. Comprehension and transformation errors were primarily related to misunderstandings of operational hierarchy, including the priority of multiplication and division over addition and subtraction, and the correct interpretation of negative signs and parentheses. Process skills errors occurred when students performed calculations inaccurately or applied operations in the wrong sequence. Encoding errors were evident when students wrote incorrect final answers that did not logically follow from preceding steps.

In-Depth Analysis of Selected Students

To illustrate representative error patterns, three students (A, B, and C) were purposively selected based on variation in their error profiles.

Student A

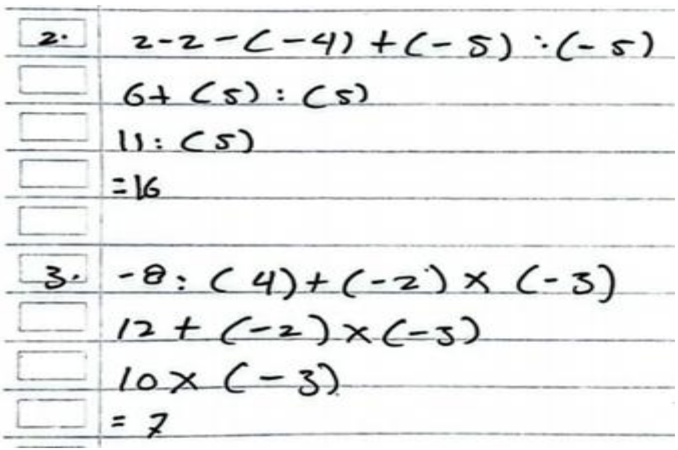

Figure 1. Student A's Response

To provide a structured analysis, Student A's responses were examined using the Newman Error Analysis (NEA) framework. The identified errors across Problems 2 and 3 are summarised in Table 3.

Table 3. Classification of Student A's Errors Based on NEA

NEA Category	Evidence from Student's Work	Description of Error	Impact on Solution
Reading Error (RE)	Added number "2" in Problem 2; changed −4 to 4 in Problem 3	Misreading/miscopying numerical symbols	Altered structure of the original expression
Comprehension Error (CE)	Ignored operational hierarchy; misinterpreted negative signs	Did not understand priority of operations and sign meaning	Led to incorrect procedural planning
Transformation Error (TE)	Added/subtracted before completing division or multiplication	Incorrect sequencing of operations	Distorted logical structure of solution
Process Skills Error (PSE)	$6 + (-5) = 11$; $10 \times (-3) = 7$	Computational inaccuracies	Produced mathematically invalid results
Encoding Error (EE)	Final answers: 16 (Problem 2), 7 (Problem 3)	Incorrect final representation	Final result inconsistent with correct answers (3 and 8)

The interview with Student A revealed that the reading errors occurred due to lack of concentration when rewriting the problem. The student stated that no external disturbance occurred; rather, the error resulted from insufficient focus during transcription. Regarding comprehension and transformation errors, the student acknowledged having previously studied integer operations but reported difficulty recalling the rules of mixed operations. The student explicitly mentioned forgetting the correct sequence of operations and the rules governing negative numbers. When asked about the computational mistakes, Student A admitted feeling uncertain while performing calculations and completing steps without verifying intermediate results. This indicates low procedural confidence and the absence of self-monitoring strategies.

The error pattern observed in Student A reflects a cascading cognitive breakdown across all five NEA stages. Initial reading inaccuracies disrupted the structural integrity of the mathematical expressions, thereby affecting comprehension and transformation processes. Misunderstanding of operational hierarchy and negative sign conventions suggests a fragile conceptual understanding rather than merely procedural weakness. Furthermore, the presence of process-skill errors indicates limited fluency in integer arithmetic, whereas encoding errors indicate inadequate reflective checking before finalising answers. The interview findings reinforce that the primary contributing factors were weak conceptual retention and reduced concentration. Overall, Student A's case illustrates how errors in early cognitive stages (reading and comprehension) can propagate to later stages (process execution and encoding), resulting in complete solution failure. This systematic progression of errors underscores the importance of strengthening conceptual understanding and metacognitive monitoring in learning mixed-integer operations.

Student B

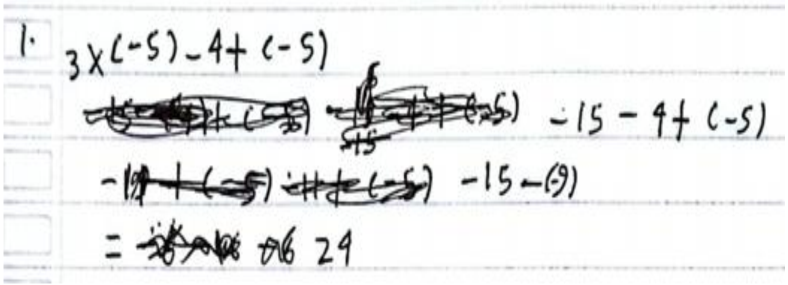

Figure 2. Student B's Response

Student B's written responses were analyzed using the Newman Error Analysis (NEA) framework. The identified errors are summarized in Table 4.

Table 4. Classification of Student B's Errors Based on NEA

NEA Category	Evidence from Student's Work	Description of Error	Impact on Solution
Comprehension Error (CE)	Rigid use of "KUKABATAKU"	Misunderstood operational hierarchy; assumed fixed sequential order	Misinterpretation of priority rules
Transformation Error (TE)	Processed $-4 + (-5)$ before completing left-to-right subtraction	Incorrect sequencing of operations	Logical distortion of solution steps
Process Skills Error (PSE)	$-15 - (-9) = 24$	Computational inaccuracy	Produced a mathematically invalid intermediate result
Encoding Error (EE)	Final answer: 24	Final result inconsistent with correct reasoning	Incorrect overall solution

Interview data revealed that Student B relied heavily on the mnemonic "KUKABATAKU" (Parentheses, Multiplication, Division, Addition, Subtraction) as a procedural guide. The student stated that multiplication must always be performed before division, and addition before subtraction, following the literal order of the acronym. This indicates a mechanical application of memorized rules rather than a conceptual understanding of principles of equal priority. The student acknowledged understanding that multiplication should be performed before addition; however, the idea that multiplication and division are of equal priority and must be processed from left to right was not fully understood. Additionally, the student reported reduced concentration during computation, which contributed to arithmetic errors.

Student B's error pattern reflects a conceptual rigidity rather than complete misunderstanding. The correct initial step ($3 \times -5 = -15$) demonstrates partial procedural knowledge. However, comprehension errors arose from overgeneralizing the mnemonic without understanding the underlying mathematical structure. The transformation error indicates difficulty translating conceptual knowledge into accurate procedural sequencing. Once the operational order was incorrectly structured, subsequent computational steps became flawed. The process skills error further suggests limited accuracy in integer arithmetic, while the encoding error reflects absence of verification before finalizing the answer. Unlike Student A, whose errors began at the reading stage, Student B's errors originated primarily at the comprehension and transformation stages. This suggests that the student's difficulty lies not in decoding the problem but in conceptualizing operational equivalence and applying left-to-right processing consistently. The interview findings reinforce that reliance on rote memorization without conceptual grounding significantly contributed to the observed error pattern.

Student C

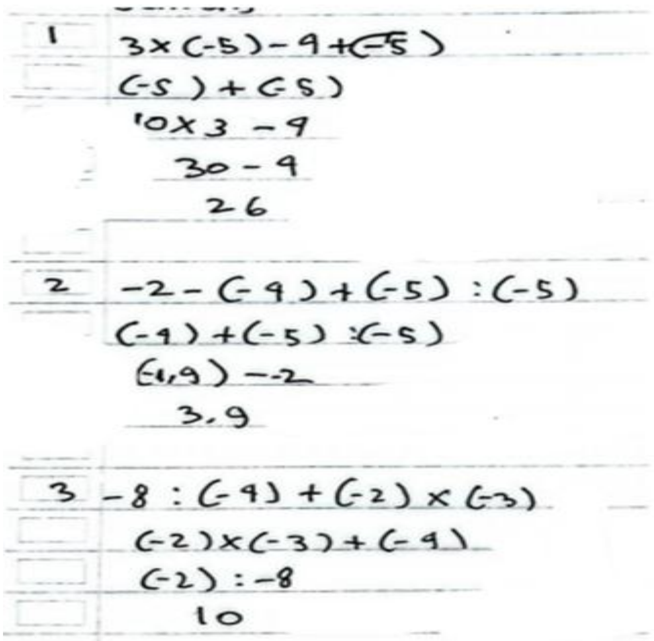

Figure 3. Student C's Response

Student C's responses were analyzed using the Newman Error Analysis (NEA) framework. The identified errors across the examined items are summarized in Table 5.

Table 5. Classification of Student C's Errors Based on NEA

NEA Category	Evidence from Student's Work	Description of Error	Impact on Solution
Comprehension Error (CE)	Treated "KUKABATAKU" as rigid sequence; assumed parentheses must be solved first	Misunderstood equal priority of operations and function of parentheses	Conceptual distortion of operational hierarchy
Transformation Error (TE)	Grouped all numbers in parentheses before other operations	Incorrect restructuring of expression	Altered procedural flow of solution
Process Skills Error (PSE)	$(-8) : (-4) = -2$	Incorrect computation of integer division	Produced invalid intermediate result
Encoding Error (EE)	Final answers inconsistent with correct solution	Incorrect representation of result	Overall solution incorrect

Interview data revealed that Student C had previously learned integer operations but had forgotten key concepts over time. The student admitted feeling confused and panicked while solving the problems. When asked about operational rules, the student explained that the mnemonic "KUKABATAKU" was applied strictly in written order (Parentheses → Multiplication → Division → Addition → Subtraction). The student believed that multiplication must always precede division and addition must precede subtraction, without recognizing that multiplication and division share equal priority and must be processed from left to right. Additionally, the student assumed that any number in parentheses must be calculated first, even when the parentheses merely indicated a negative sign rather than an operation with higher precedence. This misunderstanding directly influenced both the transformation and computational stages.

Student C's error pattern indicates a strong misconception at the comprehension stage, particularly regarding operational hierarchy and the conceptual meaning of parentheses. Unlike Student B, whose misunderstanding was procedural rigidity, Student C demonstrated a deeper conceptual misinterpretation of mathematical structure. The transformation errors indicate that the student reconstructed the mathematical expression from incorrect conceptual assumptions. This restructuring altered the intended sequence of operations, leading to flawed computational procedures. The process-skill error in dividing two negative numbers reflects an incomplete understanding of the sign rules in integer arithmetic. The interview findings highlight affective factors—such as panic and confusion—as additional contributors to error formation. Weak retention of prior knowledge combined with emotional pressure during problem-solving likely intensified the conceptual breakdown. Overall, Student C's case illustrates how a misunderstanding of fundamental operational principles can propagate through the transformation and computation stages, ultimately resulting in incorrect final encoding.

DISCUSSION

The findings of this study reveal a pervasive pattern of conceptual and procedural difficulties among Grade VII students when solving mixed-integer operation problems. The extremely low correct response rates—remaining below 15% across all items—indicate that students' understanding of mixed operations involving negative numbers is structurally fragile. This pattern aligns with research demonstrating that misconceptions in integer topics often reflect deep-seated conceptual distortions rather than isolated computational mistakes (Jeranah et al., 2025). Similar studies in mathematics education report that persistent misconceptions frequently stem from incomplete internalization of number properties and operational rules (Obunge & Adolphus, 2024). Thus, the high frequency of incorrect responses in this study suggests systemic cognitive gaps rather than superficial arithmetic errors.

The application of Newman Error Analysis (NEA) clarified that breakdowns occurred predominantly beyond the reading stage. Minimal reading errors indicate that students were generally capable of decoding numerical symbols correctly, consistent with developmental research showing that by upper elementary grades, students typically demonstrate mature digit-string recognition (Dotan et al., 2025). However, the dominance of comprehension and transformation errors suggests that students struggled to interpret the operational hierarchy and translate expressions into coherent procedural plans. This supports prior findings that difficulties in word and symbolic problems often arise during the representation and transformation stages rather than during symbol recognition (Verschaffel et al., 2020; Johar & Lubis, 2018).

The prevalence of comprehension errors highlights a misunderstanding of operational priority, particularly the equal precedence of multiplication and division, as well as addition and subtraction, which must be processed from left to right. Many students rigidly relied on the mnemonic "KUKABATAKU", applying it as a fixed sequence rather than understanding its relational equivalence. Research on conceptual versus procedural knowledge indicates that when instruction emphasizes procedural sequences without relational explanation, students develop fragmented and inflexible understanding (Rittle-Johnson et al., 2016; Keazer & Phaiah, 2023). Similarly, concept-focused instruction has been shown to produce stronger algebraic reasoning than purely procedure-focused approaches (Lee, 2025). The misuse of mnemonics in this study reflects procedural memorization that is detached from conceptual grounding, thereby limiting students' flexibility in applying operational rules.

Transformation errors further reveal students' difficulty in constructing mathematically coherent representations. Incorrect regrouping of expressions and inappropriate prioritization of operations demonstrate weaknesses in structural reasoning. Research on mathematical fluency emphasizes that procedural automation must be supported by conceptual structure to prevent systematic misapplication (Foster, 2018; Schulz, 2024). When students fail at the transformation stage, subsequent computational processes are built upon flawed representations, leading to cascading inaccuracies. This phenomenon resembles structured error propagation, where early misinterpretation amplifies downstream errors (Barrau & Bonnabel, 2020). Although originally conceptualized in computational modeling contexts, the principle of sequential error amplification provides a useful analogy for understanding cascading cognitive breakdowns in mathematics problem solving.

Process-skill errors indicate limited procedural fluency in integer arithmetic, particularly in applying the sign rules. Computational inaccuracies such as $-15 - (-9) = 24$ or $(-8) \div (-4) = -2$ demonstrate incomplete mastery of integer operations. Research on developing mathematical fluency suggests that procedural competence requires deliberate practice integrated with conceptual reasoning rather than repetitive mechanical exercises (McGee et al., 2017). Furthermore, studies examining computational errors highlight that inaccurate intermediate steps often arise from insufficient verification and limited internal monitoring (Zaslavski, 2016, 2020, 2024).

The persistence of encoding errors in this study indicates weak metacognitive monitoring. Students rarely revisited their final answers to assess plausibility or internal consistency. Metacognitive strategy research shows that self-checking, reflective evaluation, and error detection significantly enhance problem-solving accuracy (Loh & Lee, 2019; Kusaka & Ndihokubwayo, 2022). Without such strategies, students are less likely to identify inconsistencies between intermediate steps and final outcomes. The cross-case analysis of Students A, B, and C reveals distinct cognitive mechanisms underlying similar quantitative outcomes. Student A exhibited cascading breakdown beginning at the reading stage, Student B demonstrated procedural rigidity linked to mnemonic overgeneralization, and Student C showed deep conceptual misunderstanding compounded by affective disruption. These findings underscore that identical surface errors may originate from different cognitive sources. Emotional factors also played a role: several students reported panic, confusion, and forgetting prior knowledge. Research indicates that mathematics anxiety negatively affects working memory and problem-solving proficiency (Zhu et al., 2024; Guo & Liao, 2022). Thus, affective regulation interacts with conceptual understanding in shaping error patterns.

Overall, the results suggest that instruction on mixed integer operations may have prioritized rule memorization over conceptual reasoning. When operational hierarchy is taught as a memorized sequence without explicit explanation of equivalence principles, students may struggle to apply knowledge flexibly. Research comparing concept-focused and procedure-focused instruction consistently demonstrates that integrating conceptual explanations enhances both procedural fluency and long-term retention (Rittle-Johnson et al., 2016; Lee, 2025). Diagnostic frameworks such as NEA can help teachers identify specific cognitive breakdowns, enabling targeted interventions rather than attributing errors to general carelessness.

In conclusion, students' difficulties with mixed-integer operations stem primarily from conceptual misunderstandings at the comprehension and transformation stages, compounded by limited procedural fluency, weak metacognitive monitoring, and affective influences. The systematic use of Newman Error Analysis demonstrates that these errors follow identifiable cognitive sequences rather than occurring randomly. Addressing such patterns requires instructional approaches that integrate conceptual clarity, structured procedural practice, and metacognitive reflection to prevent cascading errors throughout the problem-solving process.

CONCLUSIONS

This study investigated Grade VII students' errors in solving mixed-integer operation problems using the systematic application of Newman Error Analysis (NEA). The findings reveal that students' difficulties are not random computational inaccuracies but reflect structured cognitive breakdowns occurring primarily at the comprehension and transformation stages. Although reading errors were minimal, comprehension, transformation, process skills, and encoding errors appeared at consistently high rates across all test items. These results indicate that students' understanding of operational hierarchy, particularly the equal precedence of multiplication and division and the left-to-right processing of operations, remains conceptually fragile. The cross-case analysis further demonstrates that similar surface-level errors may stem from different cognitive sources. Some students experienced cascading errors beginning at early stages, while others exhibited procedural rigidity or deep misconceptions regarding operational structure and negative number representation. Additionally, interview findings highlight the influence of weak knowledge retention, limited metacognitive monitoring, and affective factors such as confusion and anxiety. Together, these dimensions suggest that students' performance is shaped by an interaction between conceptual understanding, procedural fluency, and self-regulatory capacity. The study contributes theoretically by reaffirming the diagnostic value of stage-based error analysis in identifying precise cognitive breakdowns within mathematical problem solving. Rather than attributing students' incorrect answers to general carelessness, the NEA framework provides a structured lens for mapping error propagation across sequential problem-solving stages. Practically, the findings underscore the necessity of strengthening conceptual foundations before emphasizing procedural automation. Instruction on mixed integer operations should explicitly address the rationale behind operational hierarchy, promote relational understanding of negative numbers, and incorporate metacognitive strategies such as self-checking and justification of solution steps. Future research may extend this investigation by examining instructional interventions designed to reduce transformation and comprehension errors, exploring longitudinal retention of integer concepts, or integrating experimental designs to compare concept-focused and procedure-focused instructional approaches. Addressing these cognitive patterns systematically may prevent cascading errors and foster more stable mathematical reasoning in secondary education.

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