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Students' Mathematical Critical Thinking Processes in Solving Algebraic Problems: A Qualitative Analysis Based on Watson–Glaser Indicators

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Abstract

Mathematical critical thinking is a fundamental competence required for effective algebraic problem solving; however, many students experience difficulties that extend beyond procedural errors. This study aims to analyze junior secondary students' mathematical critical thinking processes in solving algebraic problems based on the Watson–Glaser critical thinking indicators. Employing a qualitative descriptive design, the study involved 40 seventh-grade students from a public junior high school in Indonesia. Written test data were used to describe the overall distribution of students' critical thinking levels, while in-depth qualitative analysis was conducted on three purposively selected students representing high, moderate, and low ability levels. Data were analyzed through indicator-based scoring and qualitative content analysis supported by interview data. The findings reveal that students' mathematical critical thinking ability was predominantly low, with significant difficulties in deductive reasoning and in evaluating arguments. Qualitative results indicate distinct reasoning patterns across ability levels: high-ability students demonstrated coherent reasoning but weak communication of conclusions, moderate-ability students showed fragmented and inconsistent reasoning, and low-ability students relied heavily on intuitive guessing with limited conceptual understanding. These findings highlight the need for process-oriented, differentiated instructional strategies that explicitly foster critical thinking in algebraic learning.

INTRODUCTION

The development of students' critical thinking has become a central objective of mathematics education in response to the increasing demand for higher-order cognitive skills in problem solving and reasoning. Critical thinking enables learners to analyze information systematically, justify decisions logically, and evaluate the validity of arguments across diverse learning contexts (Oyewo et al., 2022; Shanta & Wells, 2022). In mathematics classrooms, these skills are essential not only for procedural correctness but also for fostering deep conceptual understanding and reflective reasoning. Research has shown that instructional approaches emphasizing authentic tasks and inquiry-based learning significantly contribute to students' development of critical thinking by encouraging active engagement and reasoning (Dolapcioglu & Doğanay, 2022; Monrat et al., 2022). Without sufficient critical thinking skills, students tend to rely on memorization and routine procedures, limiting their ability to transfer knowledge to novel problems. Therefore, strengthening critical thinking in mathematics learning is widely regarded as a foundational requirement for meaningful and sustainable learning outcomes.

Among mathematical domains, algebra presents distinctive cognitive challenges because it requires students to engage with abstract symbols, recognize structural relationships, and construct logical generalizations. Algebraic problem solving extends beyond numerical manipulation, requiring the interpretation of conditions, the formulation of assumptions, and the evaluation of logical consistency (Burgos et al., 2024; Sun et al., 2023). Empirical studies consistently indicate that students experience persistent difficulties in algebra, particularly when solving word problems that require reasoning rather than routine procedures (Kenney & Ntow, 2024). These difficulties are often associated with limited critical thinking skills, including weak reasoning, misinterpretation of symbolic representations, and an inability to justify conclusions (Liu & Affas, 2024). As a result, students may arrive at answers through guessing or trial-and-error strategies without understanding underlying concepts. This evidence suggests that challenges in algebra learning are closely linked to deficiencies in critical thinking rather than to deficiencies in procedural knowledge alone.

Mathematical critical thinking encompasses a set of interrelated reasoning processes that support meaningful problem solving, including inference, assumption identification, deduction, interpretation, and evaluation of arguments. These processes align with broader conceptions of mathematical reasoning, which emphasize logical coherence and evidence-based justification (de Mast et al., 2023; Onoshakpokaiye, 2023). The Watson–Glaser Critical Thinking framework offers a comprehensive and well-established model for examining such processes, as it operationalizes critical thinking into clear and assessable indicators. Previous research has demonstrated the applicability of the Watson–Glaser framework in educational contexts for evaluating students' reasoning abilities systematically (Arif, 2024). In mathematics education, this framework is particularly relevant because it enables researchers to analyze how students construct, verify, and evaluate mathematical arguments, rather than focusing solely on final answers. Consequently, Watson–Glaser indicators provide a robust theoretical basis for investigating students' critical thinking in algebraic problem solving.

Although research on mathematical critical thinking has grown substantially, most existing studies predominantly emphasize quantitative outcomes, such as test scores, achievement levels, or statistical gains (Alfayez et al., 2022; Mukuka et al., 2023). While these studies offer valuable insights into overall performance, they provide a limited understanding of how students actually reason during problem-solving. In particular, few studies have qualitatively examined students' critical thinking processes at the level of specific indicators, particularly in algebraic contexts (Qurohman et al., 2025). Moreover, variations in reasoning processes among students with different ability levels remain underexplored, despite evidence that high-achieving and low-achieving students exhibit heterogeneous cognitive profiles (Prast et al., 2025; Almarashdi et al., 2023). This lack of process-oriented analysis restricts the ability to design instructional interventions that directly address students' reasoning difficulties. Therefore, there is a clear need for qualitative research to investigate how critical thinking manifests in algebraic problem solving.

Addressing this gap, the present study aims to describe students' mathematical critical thinking processes in solving algebraic problems, using the five Watson–Glaser indicators: inference, assumptions, deduction, interpretation, and evaluation of arguments. Using a qualitative descriptive approach, this study examines how students with varying ability levels engage with each indicator during problem solving. This process-oriented focus represents a key contribution beyond prior score-based assessments, offering a nuanced understanding of students' reasoning patterns and difficulties. Such insights are essential for conceptualizing critical mathematical thinking as a dynamic cognitive process rather than a static outcome (Monteleone et al., 2023).

The findings are expected to contribute theoretically by enriching models of mathematical critical thinking and practically by informing instructional strategies that promote logical reasoning, conceptual understanding, and argument evaluation in algebra learning.

METHODS

Research Design

This study employed a qualitative descriptive research design to explore students' mathematical critical thinking processes in solving algebraic problems. Qualitative descriptive research is particularly appropriate when the primary aim is to capture how learners engage in cognitive activities and construct meaning during problem solving, rather than to test causal relationships or intervention effects (Gündoğan & Öztürk, 2023). This approach enables researchers to examine students' reasoning processes as they unfold through written responses and explanations, providing rich descriptions of cognitive engagement. In mathematics education, qualitative designs have been widely used to investigate students' problem-solving and algebraic reasoning processes because they allow close attention to how students interpret tasks, apply strategies, and justify conclusions (Basir et al., 2022; Putri et al., 2022). Accordingly, the chosen design is consistent with the process-oriented focus outlined in the Introduction and supports an in-depth examination of critical thinking indicators manifested in students' algebraic problem solving.

Research Setting and Participants

The study was conducted from 14 to 16 April 2025 in Grade VII K of a public junior high school in Karawang, Indonesia, during the second semester of the 2024/2025 academic year. A total of 40 students participated in the initial stage of the research, serving as the primary data source for describing the overall distribution of mathematical critical thinking levels. To facilitate deeper exploration, three students were subsequently selected using purposive sampling, representing high, moderate, and low levels of critical thinking ability. Purposive sampling is widely recommended in qualitative research when the objective is to select information-rich cases that illuminate variations in cognitive processes (Ahmad & Wilkins, 2025). This strategy allowed the study to capture contrasts in reasoning patterns across ability levels, thereby strengthening the explanatory power of the qualitative analysis.

Instruments and Indicators

The primary research instrument consisted of a set of algebraic problem-solving tasks designed according to the Watson–Glaser Critical Thinking framework, which conceptualizes critical thinking through five indicators: inference, assumptions, deduction, interpretation, and evaluation of arguments. The Watson–Glaser framework has been extensively validated and applied in educational research as a robust model for assessing critical thinking processes (Hassan & Madhum, 2007; Zulmaulida et al., 2018). Each task was intentionally designed to elicit students' reasoning processes rather than merely their final answers, thereby enabling identification of how each indicator was demonstrated during problem solving. This indicator-based design aligns with prior studies in mathematics education that emphasize examining the reasoning structures underlying algebraic thinking (Basir et al., 2022).

Data Collection and Scoring Procedure

Students' responses were scored using an indicator-based rubric, with each Watson–Glaser indicator rated on a scale of 0–4, reflecting the degree to which the indicator was evident in the solution process. Indicator-based scoring has been shown to provide a structured and transparent means of assessing critical thinking in mathematics (Zulmaulida et al., 2018). The obtained scores were averaged and converted into a 0–100 scale using the formula: Score = (obtained score / maximum score) × 100. Based on this conversion, students' mathematical critical thinking abilities were classified into five levels—very low, low, moderate, high, and very high—following established classification criteria. This classification served both to describe overall performance patterns and to guide the selection of representative cases for further qualitative analysis.

Data Analysis

Data analysis was conducted in two complementary stages. First, a descriptive quantitative analysis was performed on the scores of all 40 students to identify the distribution of mathematical critical thinking levels. Second, an in-depth qualitative content analysis was applied to the written responses of the three selected students. This analysis focused on how each student engaged with the five Watson–Glaser indicators during algebraic problem solving. The analysis employed an indicator-based qualitative framework to systematically examine solution steps, reasoning patterns, and errors (Zeynivandnezhad et al., 2024). Such an approach enables the identification of strengths and weaknesses in students' reasoning while maintaining coherence between descriptive results and qualitative insights. To enhance trustworthiness, careful attention was given to consistency and transparency in the analytic process (Kornbluh, 2015).

Ethical Considerations

Ethical principles guiding school-based research were strictly observed throughout the study. Permission was obtained from the school authorities prior to data collection, and students' participation was voluntary. All participants were informed about the purpose of the study, and their identities were anonymized using codes to ensure confidentiality. Ethical compliance in educational research is essential to protect participants and maintain research integrity, particularly in studies involving minors (Bryan & Burstow, 2018). All collected data were used exclusively for research purposes.

RESULT

Overall Distribution of Students' Mathematical Critical Thinking Ability

The descriptive analysis of students' mathematical critical thinking ability revealed a low overall performance. The mean score of the 40 seventh-grade students was 30.375, indicating limited engagement with critical thinking processes in solving algebraic problems. The distribution of students' critical thinking levels is presented in Table 2.

Table 2. Distribution of Students' Mathematical Critical Thinking Ability

KBKM Interval	Number of Students	Percentage	Category
$0 \leq Pi \leq 24.95$	3	7.5%	Very Low
$24.95 \leq Pi \leq 41.56$	35	87.5%	Low
$41.56 \leq Pi \leq 58.35$	0	0%	Moderate
$58.35 \leq Pi \leq 75.5$	1	2.5%	High
$75.5 \leq Pi \leq 100$	1	2.5%	Very High

As shown in Table 2, the distribution of students' mathematical critical thinking ability is heavily concentrated in the low category, with 87.5% of students (n = 35) classified at this level. Only 5% of students reached the high or very high categories, and no student demonstrated a moderate level of critical thinking. This distribution indicates that the majority of students experienced considerable difficulty engaging in critical thinking when solving algebraic problems. Rather than constructing logical arguments, interpreting information accurately, or evaluating conclusions, students tended to rely on routine procedures without sufficient reasoning. These findings directly support the process-oriented concerns outlined in the Introduction, highlighting that students' algebraic difficulties are rooted in limited critical thinking rather than procedural weaknesses alone. Consequently, there is a clear and urgent need for instructional practices that deliberately emphasize the development of mathematical critical thinking skills alongside procedural competence.

Performance Across Watson–Glaser Critical Thinking Indicators

To further examine students' critical thinking performance, scores were analyzed across the five Watson–Glaser indicators. The distribution of students' performance for each indicator is summarized in Table 3.

Table 3. Distribution of Students' Performance Across Critical Thinking Indicators

Indicator	Very Low (N/%)	Low (N/%)	Moderate (N/%)	High (N/%)	Very High (N/%)
Inference	2 (5%)	26 (65%)	8 (20%)	3 (7.5%)	1 (2.5%)
Assumptions	0 (0%)	30 (75%)	8 (20%)	1 (2.5%)	1 (2.5%)
Deduction	1 (2.5%)	35 (87.5%)	3 (7.5%)	1 (2.5%)	0 (0%)
Interpretation	3 (7.5%)	29 (72.5%)	6 (15%)	1 (2.5%)	1 (2.5%)
Evaluation of Arguments	2 (5%)	36 (90%)	0 (0%)	1 (2.5%)	1 (2.5%)

Overall, the results indicate that all five indicators were dominated by the low and very low categories. The weakest performance was observed in the evaluation of arguments and deduction, where 90% and 87.5% of students, respectively, fell into the low category. These findings suggest that students encountered particular difficulties in evaluating the validity of arguments and in drawing logically consistent conclusions from given premises—two core components of mathematical critical thinking. This pattern aligns with the descriptive results and reinforces the need for learning experiences that emphasize reasoning, justification, and the evaluation of arguments in algebra.

Qualitative Analysis of Students' Critical Thinking Processes

To gain deeper insight into students' reasoning processes, three students were purposively selected for in-depth qualitative analysis to represent varying levels of mathematical critical thinking ability, namely high (S-1), moderate (S-2), and low (S-3), as summarized in **Table 4**. This selection was intended to capture contrasts in how students with different proficiency levels engage with algebraic problems and demonstrate the indicators of critical thinking. By examining representative cases across these categories, the qualitative analysis provides a more nuanced understanding of students' reasoning patterns, including their strengths, difficulties, and tendencies in interpreting problems, constructing arguments, and drawing conclusions.

Table 4. Selected Students for Qualitative Analysis

Student Initial	Code
KP	S-1
MZ	S-2
RR	S-3

Student S-1 (High Ability)

The written responses of S-1 indicate a generally strong command of algebraic concepts and logical reasoning. In Questions 1 and 2, S-1 correctly applied algebraic operations and demonstrated an accurate understanding of the given information. However, in Question 1, a misinterpretation of the problem statement resulted in an incorrect final conclusion, revealing a weakness in the inference indicator despite correct procedural calculations (Figure 1). In contrast, Question 2 shows that S-1 identified and formulated appropriate assumptions to support the solution process.

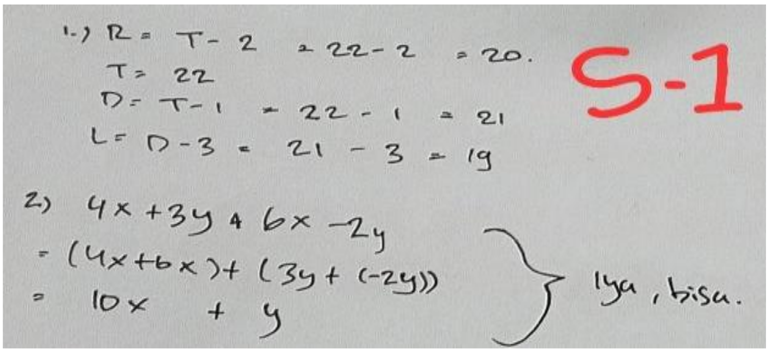

Figure 1. S-1's responses to Questions 1 and 2.

In Questions 3 and 4, S-1 demonstrated effective deductive reasoning and interpretation by systematically assigning variables and extracting implicit information from the problem context, resulting in coherent, logically structured solutions (Figure 2). Furthermore, in Question 5, S-1 exhibited the ability to evaluate arguments by substituting given values and assessing the validity of the mathematical claim, although minor interpretive inaccuracies were still evident (Figure 3).

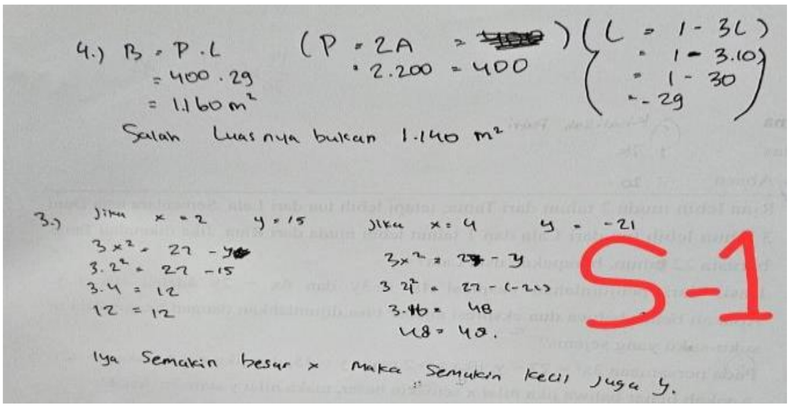

Figure 2. S-1's responses to Questions 3 and 4.

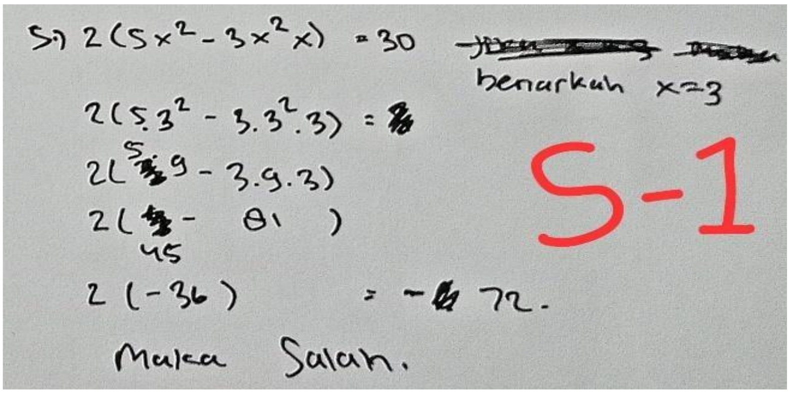

Figure 3. S-1's response to Question 5.

Interview data, summarized in Table 5, further confirm that S-1 possessed strong logical reasoning skills but had difficulty articulating conclusions explicitly in writing. Overall, S-1 demonstrated a high level of mathematical critical thinking, with remaining weaknesses primarily related to careful reading of problem statements and the clear communication of conclusions.

Table 5. The Interview Findings for Student S-1

Critical Thinking Indicator	Interview Evidence (Excerpt)	Empirical Interpretation
Inference	"I usually rely on logic, but sometimes I don't know how to write the conclusion clearly."	S-1 is able to draw conclusions mentally, but has difficulty expressing final conclusions explicitly in written form.
Assumptions	"I looked at the variables first and assumed what x and y represented."	S-1 demonstrates appropriate assumption-making based on algebraic structures and problem conditions.
Deduction	"If the variables fit the conditions, then the result must be correct."	Indicates consistent use of deductive reasoning by linking premises to conclusions logically.
Interpretation	"At first I misunderstood the question, but after rereading it, I corrected my answer."	Shows the ability to reinterpret information and revise reasoning when initial understanding is inaccurate.
Evaluation of Arguments	"After substituting the value, I checked whether the statement was true or false."	Reflects the ability to evaluate the validity of mathematical arguments through verification and substitution.
Communication of Reasoning	"I find it hard to explain what I think in writing."	Reveals a weakness in communicating reasoning and conclusions, despite strong internal logical processing.

Student S-2 (Moderate Ability)

The written responses of S-2 indicate a partial understanding of algebraic concepts accompanied by basic, yet inconsistent, reasoning skills. In Question 1, S-2 was able to apply algebraic operations correctly and obtained the appropriate numerical result; however, the absence of an explicit concluding statement reveals a limitation in the inference indicator. Although the computational process was accurate, the reasoning was not finalized by a clear, logically stated conclusion, suggesting incomplete critical thinking (Figure 4). Similarly, in Question 2, S-2 correctly identified relevant assumptions underlying the algebraic expressions, but these assumptions were expressed briefly and without sufficient justification, indicating limited elaboration of the underlying reasoning process (Figure 5).

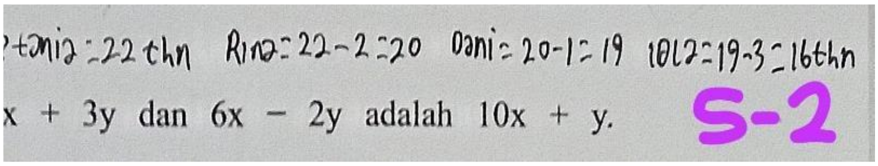

Figure 4. S-2's response to Question 1.

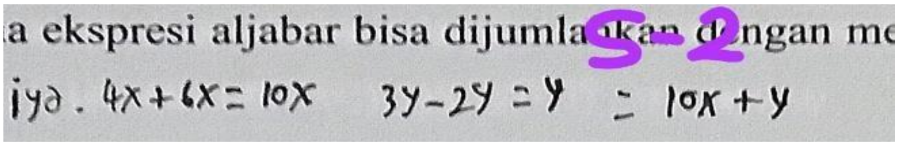

Figure 5. S-2's response to Question 2.

Difficulties became more pronounced in Question 3, where S-2 misinterpreted the problem context and applied inappropriate algebraic procedures, resulting in incorrect deductions. This error reflects a weakness in the deduction indicator, as conclusions were drawn without a logical connection to the given premises (Figure 6). In contrast, Question 4 demonstrates that S-2 adequately interpreted the problem by extracting relevant information and applying it correctly. Furthermore, in Question 5, S-2 demonstrated a reasonable ability to evaluate arguments under the given conditions. Nevertheless, in both questions, S-2 again failed to articulate a concluding statement, indicating inconsistency in communicating the reasoning and verifying the final results.

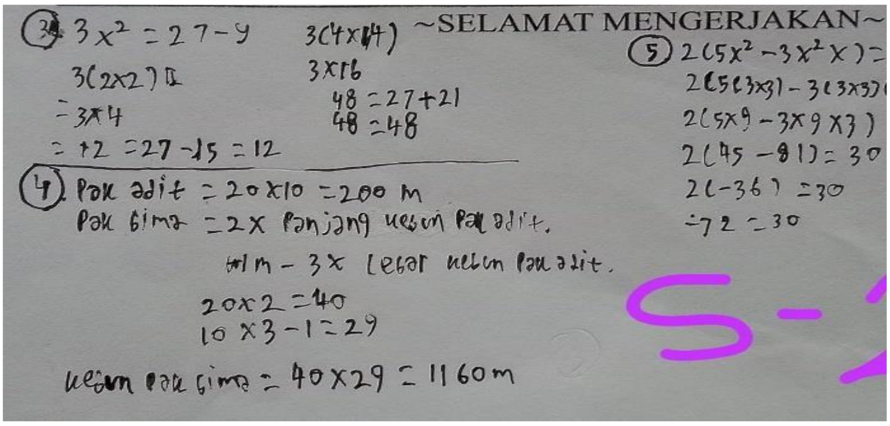

Figure 6. S-2's responses to Questions 3, 4, and 5.

Interview data, summarized in Table 6, further reveal that S-2 relied heavily on trial-and-error strategies, expressed uncertainty regarding the correctness of answers, and tended to avoid drawing explicit conclusions due to low confidence. These findings suggest that, although S-2 possesses emerging reasoning abilities and basic conceptual understanding, the critical thinking process remains fragmented and unsystematic. Overall, S-2 demonstrated a moderate level of mathematical critical thinking, characterized by partial reasoning competence that has not yet developed into well-structured, logically articulated, and confidently communicated problem-solving processes.

Table 6. The Interview Findings for Student S-2

Critical Thinking Indicator	Interview Evidence (Excerpt)	Empirical Interpretation
Inference	"I'm often unsure whether my answer is correct, so I don't write a conclusion."	S-2 avoids drawing explicit conclusions due to low confidence, despite reaching correct results in some cases.
Assumptions	"I just separate the variables that can be combined."	Indicates recognition of basic assumptions, but without deeper justification or explanation.
Deduction	"I tried different values because I wasn't sure which one was right."	Reflects reliance on trial-and-error rather than systematic deductive reasoning.
Interpretation	"I understand what the problem is asking after I read it again."	Shows adequate interpretation ability, although initial understanding is often uncertain.
Evaluation of Arguments	"If the calculation works, I think the answer is okay."	Evaluation is based on procedural correctness rather than logical validation of arguments.
Confidence and Communication	"I'm afraid my explanation is wrong, so I don't write much."	Reveals limited confidence and weak communication of reasoning, affecting the clarity of conclusions.

Student S-3 (Low Ability)

The written responses of S-3 reveal substantial difficulties across all indicators of mathematical critical thinking. Across Questions 1–5, S-3's responses were largely incomplete, inconsistent, or based on guessing rather than systematic reasoning (Figure 7). In Question 1, S-3 provided a numerical answer without demonstrating an understanding of the problem context, indicating a weakness in the inference indicator. Similarly, in Question 2, S-3 failed to identify relevant assumptions, suggesting an inability to recognize underlying algebraic structures. More pronounced difficulties were observed in Questions 3 and 4, where S-3 was unable to apply deductive reasoning or interpret the given information meaningfully. Solution steps were either absent or unrelated to the problem requirements, reflecting a lack of logical connection between premises and conclusions. In Question 5, S-3 did not attempt to evaluate the given argument and provided a response without justification, further indicating an undeveloped evaluation of arguments indicator.

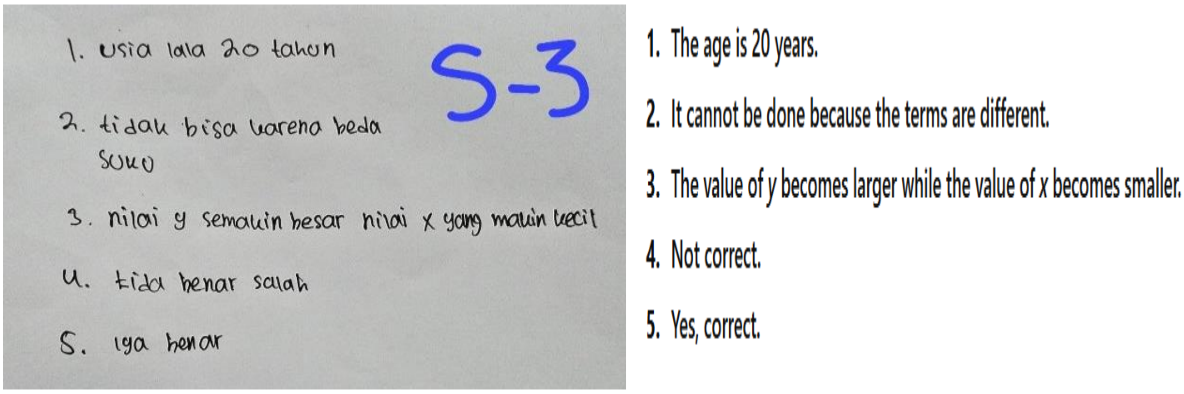

Figure 7. S-3's responses to Questions 1–5.

Interview data, summarized in Table 7, corroborate the findings from the written test, revealing that S-3 lacked a fundamental understanding of algebraic concepts and was unable to explain solution steps or justify answers. S-3 frequently expressed confusion when interpreting problem statements and relied on intuition rather than logical reasoning when responding. Conclusions were either omitted or stated without logical grounding. Overall, S-3 demonstrated a low level of mathematical critical thinking, characterized by intuitive guessing, limited conceptual understanding, and the absence of systematic reasoning. These findings highlight the need for targeted instructional support and structured scaffolding to assist students like S-3 in developing foundational algebraic understanding and critical thinking skills.

Table 7. The Interview Findings for Student S-3

Critical Thinking Indicator	Interview Evidence (Excerpt)	Empirical Interpretation
Inference	"I just wrote the answer. I don't really know how to conclude it."	S-3 provides answers without forming or articulating logical conclusions, indicating undeveloped inference skills.
Assumptions	"I don't really understand which variables can be used."	Shows an inability to identify or formulate relevant assumptions in algebraic problems.
Deduction	"I just tried to answer without knowing the steps."	Reflects reliance on intuitive guessing rather than deductive reasoning based on given premises.
Interpretation	"I don't really understand what the question is asking."	Indicates difficulty interpreting problem statements and extracting relevant information.
Evaluation of Arguments	"I don't know whether it's right or wrong."	Demonstrates inability to evaluate the validity of mathematical arguments or solutions.
Conceptual Understanding	"I don't really understand algebra."	Confirms limited foundational understanding of algebraic concepts.

Communication of Reasoning	"I don't know how to explain my answer."	Reveals inability to communicate reasoning processes and justify answers verbally or in writing.

DISCUSSION

The findings of this study indicate that the overall level of students' mathematical critical thinking in solving algebraic problems was predominantly low. This result is consistent with previous studies reporting that students' difficulties in algebra are not solely attributable to procedural weaknesses but are more fundamentally associated with limited reasoning, interpretation, and argumentation skills (Harti & Agoestanto, 2019; Cahyono et al., 2019). The dominance of the low category across almost all indicators confirms that algebraic problem solving requires more than computational competence; it demands the ability to interpret information, formulate assumptions, and justify conclusions logically. Similar patterns have been reported in studies of algebraic problem solving that emphasize process-oriented reasoning over routine calculation (Widodo et al., 2020; Nurhidayati & Setyaningsih, 2024). These findings reinforce the process-oriented perspective emphasized in the Introduction, highlighting critical thinking as a central challenge in algebra learning.

Analysis across the Watson–Glaser indicators reveals that deduction and evaluation of arguments were the weakest aspects of students' critical thinking. Most students struggled to determine whether conclusions logically followed from given premises or to assess the validity of mathematical statements. This finding aligns with research demonstrating a strong link between deductive reasoning and mathematical performance, particularly in algebraic contexts (Morsanyi et al., 2018). From a cognitive perspective, deduction and evaluation are higher-level reasoning processes that require explicit coordination between premises and conclusions, a skill that many students fail to develop adequately (Goswami, 2010; Sherry, 2006). The limited performance in inference and interpretation further suggests that students were often unable to transform problem information into coherent mathematical representations, a difficulty that has been widely reported in algebra learning research (Widodo et al., 2020).

The qualitative analysis provides deeper insight into these quantitative patterns. Student S-1, categorized as high ability, demonstrated coherent reasoning across all indicators, confirming that strong conceptual understanding enables students to engage effectively in deductive reasoning and argument evaluation. This finding is consistent with studies showing that high-achieving students tend to construct more structured and logically consistent solution strategies (Aguilar et al., 2016; Almarashdi et al., 2025). However, even S-1 experienced difficulty in articulating conclusions explicitly, indicating that communication of reasoning is a distinct component of critical thinking that does not automatically develop alongside internal logical competence. Previous research has similarly noted that students may possess strong internal reasoning while struggling to externalize their thinking in written mathematical explanations (Widodo et al., 2020).

In contrast, Student S-2 exhibited emerging critical thinking abilities characterized by partial understanding and inconsistent reasoning. Although S-2 was able to perform basic algebraic operations and interpret information in certain tasks, the frequent absence of conclusions and reliance on trial-and-error strategies reflect undeveloped deductive structures and low confidence. This pattern corresponds to findings that moderately achieving students often occupy a transitional stage of reasoning development, in which procedural knowledge exists but has not yet been integrated into systematic problem-solving strategies (Jordan & McDaniel, 2014). Without targeted instructional support, students at this level may fail to progress toward higher levels of critical thinking because uncertainty and lack of confidence impede the consolidation of reasoning processes (O'Shea et al., 2017).

Student S-3's performance highlights the challenges faced by learners with low levels of mathematical critical thinking. The absence of logical reasoning, inability to interpret problems, and reliance on intuitive guessing indicate a lack of foundational understanding of algebra. Interview data further reveal that S-3 was unable to justify answers or explain reasoning, underscoring the close relationship between conceptual knowledge and critical thinking processes. Similar findings have been reported in studies showing that low-achieving students often rely on surface-level strategies and guessing when confronted with cognitively demanding algebraic tasks (Parsons, 2016; Woolf et al., 2010). These results suggest that students at this level require intensive and structured scaffolding that simultaneously targets conceptual understanding and reasoning skills.

The results demonstrate that students' mathematical critical thinking abilities vary qualitatively across ability levels rather than merely quantitatively. High-ability students tend to exhibit structured reasoning, with minor weaknesses in communication; moderate-ability students show fragmented reasoning and uncertainty; and low-ability students struggle fundamentally across all indicators. This qualitative variation supports earlier research emphasizing that differences in mathematical performance are closely linked to differences in reasoning processes rather than to computational skill alone (Elliott et al., 2001; Harti & Agoestanto, 2019). Pedagogically, these findings imply that mathematics instruction should incorporate learning activities that explicitly foster critical thinking processes, such as inference, deduction, and the evaluation of arguments. Problem-based and inquiry-oriented tasks that require students to articulate assumptions, justify conclusions, and evaluate alternative solutions have been shown to enhance critical thinking and algebraic reasoning (Harti & Agoestanto, 2019; Elliott et al., 2001). Moreover, structured questioning, guided reflection, and scaffolded support are essential for helping moderate- and low-achieving students develop confidence and systematic reasoning skills, thereby reducing reliance on trial-and-error and intuitive guessing.

CONCLUSIONS

This study concludes that junior secondary students' mathematical critical thinking in solving algebraic problems remains predominantly low, characterized by fundamental weaknesses in reasoning processes rather than by procedural competence alone. Analysis based on the Watson–Glaser indicators reveals that deduction and evaluation of arguments are the most challenging aspects, indicating students' limited ability to justify conclusions logically and to assess the validity of mathematical statements. These findings confirm that algebraic problem solving requires higher-order cognitive engagement, including interpretation, assumption formulation, and logical verification, which many students have not yet adequately developed. The qualitative findings further demonstrate that students' critical thinking abilities differ qualitatively across ability levels. High-ability students exhibit coherent reasoning and strong conceptual understanding but still experience difficulties in articulating conclusions explicitly. Moderately able students exhibit emerging reasoning skills that remain fragmented and inconsistent, often accompanied by uncertainty and reliance on trial-and-error strategies. In contrast, low-ability students struggle fundamentally with interpreting problems, constructing logical arguments, and justifying answers, relying instead on intuitive guessing. This variation highlights that differences in students' performance are rooted in distinct reasoning patterns rather than merely in achievement scores.

Pedagogically, these results emphasize the need for mathematics instruction that explicitly targets the development of critical thinking processes alongside procedural fluency. Instructional practices should prioritize activities that require students to explain reasoning, formulate and test assumptions, draw logical conclusions, and evaluate arguments. Differentiated scaffolding is particularly important for supporting moderate- and low-ability students in developing foundational algebraic understanding and systematic reasoning skills. Overall, strengthening mathematical critical thinking through process-oriented learning approaches is essential for improving students' algebraic problem-solving competence and for fostering deeper, more meaningful mathematics learning.

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